



Brain Ticklers

RESULTS FROM SPRING 2011

Perfect

Couillard, J. Gregory	IL A '89
*Kimsey, David B.	AL A '71
*Mayer, Michael A.	IL A '89
Slegel, Timothy J.	PA A '80
*Spong, Robert N.	UT A '58
*Stout, Roger P.	AZ B '77
*Strong, Michael D.	PA A '84

Other

Alexander, Jay A.	IL Γ '86
Aron, Gert	IA B '58
Bachmann, David E.	MO B '72
*Beaudet, Paul R.	Father of member
Bernacki, Stephen E.	MA A '70
Bertrand, Richard M.	WI B '73
Brule, John D.	MI B '49
Conway, David B.	TX I '79
*Fenstermacher, T. Edward	MD B '80
Glaser, Anton	Non-member
Grewal, Rashi	NJ Γ '09
Harter, Eamonn T.	ID Γ '06
*Harvey, Arthur J.	OH A '83
*Jones, John F.	WI A '59
Oliver, Christopher R.	AL E '08
Prince, Lawrence R.	CT B '91
Quintana, Juan S.	OH Θ '62
Rasbold, J. Charles	OH A '83
Robillard, David J.	MD Γ '88
*Routh, Andre G.	FL B '89
*Schmidt, V. Hugo	WA B '51
Sentman, Mark H.	CA E '86
Sentman, Andrew	Son of member
Silver, Robert E.	NY P '80
Stadlin, Walter O.	NJ Γ '52
Stribling, Jeffrey R.	CA A '92
Stumpf, Kenneth A.	MI Δ '73
Summerfield, Steven L.	MO Γ '85
Sutor, David C.	Son of member
Van Wyk, Rogell	IN A '59
*Voellinger, Edward J.	Non-member
Wills, Kenneth J.	MT A '84

* Denotes correct bonus solution

SPRING REVIEW

The Spring Ticklers appear to have been more difficult than usual, with only a few perfect scores. Two regular problems (Number 2 about a spider chasing an ant and Number 4 about card arrangements) had a lower percentage of correct answers than the Bonus problem.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter BENT. Meanwhile, here are the answers:

1 The points awarded to Al, Bob, Carl, Don, and Ed on each question of the five-question math test are shown in the following table.

Question	Points Awarded					Total points each student
	1	2	3	4	5	
Al	2	10	10	10	2	34
Bob	0	7	0	10	2	19
Carl	2	10	2	10	7	31
Don	10	2	10	10	10	42
Ed	0	0	0	2	7	9
Total each question	14	29	22	42	28	

To arrive at this table, note that the totals of the rows and the columns can only be expressed as a sum of the numbers 10, 7, 2 and 0; possibilities are: $9 = 7 + 2$; $14 = 10 + 2 + 2 = 7 + 7$; $19 = 10 + 7 + 2$; $22 = 10 + 10 + 2$; $29 = 10 + 10 + 7 + 2$; $31 = 10 + 7 + 7 + 7 = 10 + 10 + 7 + 2 + 2$; $34 = 10 + 10 + 10 + 2 + 2 = 10 + 10 + 7 + 7$; $42 = 10 + 10 + 10 + 10 + 2$. Because Q4 was so easy and Ed only got a total of 9 points, Ed must have been the one who got a 2 on Q4; therefore, Column 4 must be (10, 10, 10, 10, 2). Since Ed scored higher than Bob on Q5, Row 5 must be (0, 0, 0, 2, 7). Note that if the answer to Q2 were 17, then three 10s would be awarded, the sum of which is larger than 29; therefore, the second column must be (10, ?, 10, ?, 0) with the unknowns being 2 and 7.

Note that if the answer to Q3 were 5, then Bob would have scored two 10s, which sum to more than 19, so '5' is not the answer to Q3. Since Row 3 sums to 22, the correct answer must be '11,' and Column 3 is (10, ?, ?, 10, 0) with unknowns 0 and 2. Since Row 1 sums to 34 and already contains three 10s, the other two scores must be 2, so Row 1 is (2, 10, 10, 10, 2). Column 1 sums to 14 and must contain a 10, but the 10 can't be in Rows 2 or 3 because the row sums won't work. Therefore, Don got the 10 on Q1.

Next, note that for Column 1 to sum to 14 and Row 3 to sum to 31, Carl must have scored a 2 on Q1. For Column 3 to sum to 22 and Row 3 to sum to 31, Carl must have scored a 2 on Q3. This means that Carl scored a 7 on Q5. Since Ed got more credit for Q5 than Bob, Bob must have scored a 2 on Q5 and a 7 on Q2. Finally, the only remaining score to be determined is Don's score on Q2, which must be a 2.

2 Stated in base 11, the simultaneous cryptic additions decode as:

USA + FDR = WPA : $5a4 + 320 = 914$
 USA + WPA = PARK : $5a4 + 914 = 1408$,
 where 'a' stands for the digit 10.

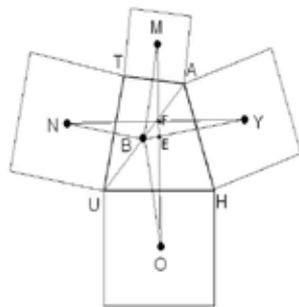
3 In a game of 7-card stud with deuces wild, the probability of getting 5-of-a-kind is $7,622/1,672,307 \approx 0.004558$. There are $52!/(45!7!) = 133,784,560$ possible 7-card hands. Consider the probability of getting one deuce and four cards of the same denomination. There are 12 possibilities for the 5-of-a-kind card (J, Q, K, A, or 3 through 10). There are four possibilities for the deuce (club, diamond, heart, or spade), and $44(43)/2$ possibilities for the two extra cards (which can't be a deuce). This makes $4(12)(44)(43)/2 = 45,408$ ways to get 5-of-a-kind using only one deuce. The following table shows the cases that must be considered. The probability is the total ways to get 5-of-a-kind divided by the number of possible hands: $609,760/133,784,560 = 7,622/1,672,307 \approx 0.004558$.

No. of 2s	5 of a Kind Cards	No. of Other Cards	Formula	Value
1	4K	2	$4(12)(44)(43)/2$	45,408
2	3K	2	$6(12)(4)(44)(43)/2$	272,448
2	4K	1	$6(12)(44)$	3,168
3	1Pr	2	$4(12)(6)(44)(40)/2$	253,440
3	2Pr	0	$4(12)(11)(6^2)/2$	9,504
3	3K	1	$4(12)(4)(44)$	8,448
3	4K	0	$4(12)$	48
4		3	$48(47)(46)/6$	17,296
Total ways to get 5-of-a-kind				609,760

4 To divide 26 coins, 10 heads and 16 tails, into two groups with an equal number of heads in each group while blindfolded, first divide the coins into two groups with 10 coins in one group and 16 coins in the other group. Then, flip over the ten coins in the 10-coin group. To see how this works, let H be the number of heads in the 10-coin group; then, $10 - H$ is the number of heads in the 16-coin group. When the 10-coin group is flipped, the number of heads will be $10 - H$, the same as the 16-coin group.

5 The number $z = i$ to the i th to the i th power ad infinitum approaches the limit $r(\cos\theta + i\sin\theta)$, where $r = 0.567555$ and $\theta = 0.688454$ radians. To see this, substitute the expression for z into z to get $z = i^z$.

Next, express i as $e^{i\pi/2}$ to get $z = e^{i\pi z/2}$. Let $z = re^{i\theta} = r(\cos\theta + i\sin\theta)$, and substitute into the modified equation for z , yielding $re^{i\theta} = e^{i\pi r(\cos\theta + i\sin\theta)/2} = e^{-\pi r\sin\theta/2}e^{i\pi r\cos\theta/2}$. From this, we get two simultaneous equations in r and θ : $r = e^{-\pi r\sin\theta/2}$ (or $\ln r = -\pi r\sin\theta/2$) and $\theta = \pi r\cos\theta/2$. Solving yields $r = 0.56755$ and $\theta = 0.68845$ radians. Solution by trial and error is easy using a spreadsheet; pick r and calculate $\theta = \sin^{-1}(-2\ln r/\pi r)$. Then, calculate $r = 2\theta/\pi\cos\theta$; if the r 's don't match, pick a new r .



BYN equals angle BOM. Let E be the point where MO and BY cross. Since $\text{OBY} = 90^\circ$, then $\text{BOE} + \text{BEO} = 90^\circ$. Let F be the point where NY and MO cross. Since $\text{MEY} = \text{BEO}$ and $\text{BYN} = \text{BOM}$, then $\text{BYN} + \text{MEY} = 90^\circ$. Therefore, $\text{OFY} = 90^\circ$ and, hence, $\text{YFM} = 90^\circ$. Therefore, NY is perpendicular to MO.

NEW FALL PROBLEMS

Bonus. The 2x4 map, numbered 1-8 on one side and a-h on the other side, can be folded into 40 different final configurations that have the “1” on the top. The configurations can be found using snake diagrams. (See W. F. Lunnon, “Multi-dimensional map-folding,” *The Computer Journal* 14(1):75-80, February 1971. comjnl.oxfordjournals.org/contact/14/1/75.full.pdf+html.) The 40 configurations, listed in alphanumeric order of the upper surfaces, are:

- | | | |
|-----------|-----------|-----------|
| 13d8ge6b, | 13db68ge, | 13g8de6b, |
| 13ge68db, | 18d3ge6b, | 18g3db6e, |
| 1b3d8g6e, | 1b3g6e8d, | 1b3g8d6e, |
| 1b68g3de, | 1b6d8g3e, | 1b6e8g3d, |
| 1b6ed8g3, | 1b6eg3d8, | 1b6eg8d3, |
| 1b6g3d8e, | 1b6g8d3e, | 1b8d3g6e, |
| 1bd3g86e, | 1bd86eg3, | 1d3b6g8e, |
| 1d3g8e6b, | 1d8e6g3b, | 1d8g3b6e, |
| 1e3d8g6b, | 1e3g8d6b, | 1e68g3db, |
| 1e6b3d8g, | 1e6b3g8d, | 1e6b8d3g, |
| 1e6bd3g8, | 1e6d8g3b, | 1e6g3d8b, |
| 1e6g8d3b, | 1e8d3g6b, | 1e8g6b3d, |
| 1ed3g86b, | 1eg86bd3, | 1g3d8b6e, |
| 1gd3b6e, | | |

Double Bonus. Van Aubel’s Quadrilateral Theorem states that the lines connecting the midpoints of squares constructed on opposite sides of any quadrilateral are equal in length and mutually perpendicular. Refer to the diagram for the proof. Construct a square on each side of the quadrilateral UTAH with centers N, M, Y, and O.

Construct UA and find the midpoint B. It can be shown that $\text{BN} = \text{BM}$, $\text{BY} = \text{BO}$, and that angles NBM and OBY are right angles (see aguite.homestead.com/files/triangle_squares1.html). Therefore, triangle NBY is congruent to triangle MBO by side-angle-side congruence. So, $\text{NY} = \text{MO}$ and angle

ask, “Is Ann’s number 11?” After receiving an honest yes or no answer on the monitor from the moderator, the questioner states aloud, either “I know my number” or “I don’t know my number.” The following conversation ensues:

- Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I don’t know my number.”
 Ann: “I don’t know my number.”
 Beth: “I know my number.”

What are Ann’s and Beth’s numbers?
 —adapted from

All-Star Mathlete Puzzles by
Dr. Richard I. Hess, CA B ’62

1 Joe the butcher received a bill in the mail. Unfortunately it had gotten wet, and the first and last digits were unreadable. What was legible was:

72 turkeys \$_67.9_

How much was the bill, and what is the cost of one turkey assuming they all cost the same?

—*The Stanford Mathematics Problem Book*, by G. Polya and J. Kilpatrick

2 Determine the equation of a parabola that passes through the four points (x_i, y_i) : (0, -1), (-1, 0), (1, 0), and (0, 3).

—**Craig K. Galer, MI A ’77**

3 Two perfect logicians, Ann and Beth, have each been assigned a number by a moderator. Neither woman knows either of the numbers, but they do know that the numbers consist of two positive odd integers that are either the same or differ by 2 and that, if the two integers are different, then the larger one is assigned to Beth. Ann and Beth sit in front of computer monitors, arranged so that each can see only her own monitor. They take turns typing in questions of the form, “Is Y’s number X?” about the other’s number, where X is a positive odd integer. For example, Ann could ask, “Is Beth’s number 7?,” and Beth could

4 Numbers b and c are chosen at random from the range of real numbers bounded by $-q$ and q . Express the probability, as a function of q , that the quadratic equation, $x^2 + bx + c = 0$, will have complex roots.

—**William S. Alderson, MI E ’43**

5 A uniform billiard ball of diameter D rolls without slipping along a pool table toward a cushion. What height of cushion creates a rebound that maintains rolling-without-slipping motion?

—**Jeffrey R. Stribling, CA A ’92**

Bonus. A geosynchronous orbit is a circular orbit with an orbital period of 24 hours. These orbits, typically used by the equatorial *stationary* class of communications satellites, have a radius R_s equal to about 26,300 miles. Another class of satellites, the Russian Molniya class, employ inclined, highly eccentric orbits characterized by a 12-hour orbital period. Assuming that the closest approach to the earth’s surface of a Molniya orbit is 100 miles to avoid burning in the atmosphere, for what range of eccentricities is the apogee greater than R_s ?

—*Technology Review*

(Continued on page 36.)

The Executive Council met in Knoxville, TN, on June 10-11, 2011.

The Council enthusiastically voted two TBPI Superior Service Awards and a Resolution of Appreciation to retiring advisors of the collegiate chapters.

Kevin A. Davis, *CA A '08*, Diana Hasegan, *PA E '10*, and Ellen S. Styles, *AL A '85*, were appointed as Engineering Futures Facilitators to terms ending June 2014. Councillor Norman Pih reported on the meeting of the Engineering Futures Planning Committee in Phoenix, AZ, on May 21. The Council re-appointed Dr. Dennis J. Tyner and Scott V. Eckersall to terms ending June 30, 2012, and Nancy F. Gray, Cheryl Cheng, and J.P. Blackford to the EFPC to terms ending June 30, 2014.

The Council re-appointed D. Stephen Pierre Jr., *AL E '90*, as Director of Fellowships, Edward J. D'Avignon, *NY B '88*, as Director of Rituals, and Russell W. Pierce, *WA A '70*, as Director of Engineering Futures to terms ending June 2015.

The Council re-appointed Dr. Sally J. Steadman and Barry L. Frost III to the Advisor Recruitment & Development Committee to terms ending June 30, 2012, and June 30, 2013, respectively and granted a TBPI Resolution of Appreciation to Dr. Robert E. Efimba, P.E., for five years of service to the committee.

ED/ST-designate Curtis D. Gomulinski, member of the 2011 Laureate Selection Committee reported that his group had named five TBPI Laureates from 11 nominees. The selections of Richard G. Higgins, *ME A '79*, as the 2011 TBPI Distinguished Alumnus and of Dr. William R. Goodin, *CA E '75*, as the 2011 TBPI-McDonald Mentor were accepted.

The Council confirmed the acceptance of Mr. Gomulinski as ED/ST-designate and accepted the report summary of the selection process.

Councillor J.F.K. Earle, Ph.D., P.E., reported on recent progress in the TBPI K-12 MindSET Program and a teacher-training session to be conducted by the Florida Alpha Chapter on June 13.

The Council reviewed and modified plans for the 2011 Convention to be held in Indianapolis, IN, on October 27-29. Proposals to locate the Headquarters permanently in Knoxville, TN, and to modify the retirement provisions for the Secretary-Treasurer were reviewed and will be sent to the Convention. The Council appointed a permanent Convention Chair and a Parliamentarian for the 2011 annual meeting.

The Council reviewed final arrangements for the June 11 meeting of national officers and directors. Councillors reported on their participation at District Conferences.

The Council reviewed the preliminary petitions for new collegiate chapters from the local engineering honor societies at Penn State Erie and the College of New Jersey, and Vice President Solange C. Dao, P.E., and Dr. Earle were named to represent the Council on inspection visits on September 8 and 22, respectively. A request from the University of Arkansas at Little Rock to be considered eligible to establish a chapter was reviewed.

President Larry A. Simonson reported on the discussions of the TBPI Vision Development Group at its meeting in New York City on April 15. The Council heard plans for the meeting of the Trust Advisory Committee on July 14 in New York City. The Council disbanded the Financial Development

IRA ROLLOVERS REINSTATED!

YOU SENIOR TAU BATES CAN ONCE AGAIN MAKE TAX-FREE DONATIONS TO TBPI for a limited time from your IRAs if you are **age 70½ or older**. A recent law allows tax-free charitable donations from an IRA under simple conditions! You must act before December 31, 2011.

This is good news for anyone contemplating making a major gift to TBPI or another qualified charity. Under the Tax Relief, Unemployment Insurance Reauthorization and Job Creation Act of 2010, any donor age 70½ or older may contribute up to \$100,000 annually from an IRA account and avoid federal-tax consequences. The charitable gift amount can count against the donor's required minimum distribution (RMD).

When the amount withdrawn from the IRA account is paid directly to the charity, it is not counted as federally taxable income. However, because the gift would be excluded from income, it may not be included as a charitable deduction on a federal tax return. These rules will apply to gifts made through December 31, 2011.

BRAIN TICKLERS

(Continued from page 35.)

Double Bonus. What are the last seven digits of 7,777,777,777?

—Adapted from Colin Singleton in *New Scientist*

Send your answers to any or all of the Brain Ticklers to: **Curt Gomulinski, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697**, or email plain text only to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT in December. The method of solution is not necessary. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **F.J. Tydeman, CA A '73**; **D.A. Dechman, TX A '57**, and the columnists for this issue,

—**J.L. Bradshaw, PA A '82**, and
Dr. H.G. McIlvried III, PA I '53.

ment Committee and voted TBPI Resolutions of Appreciation to Sherry D. Jennings-King and James W. Johnson Jr.

Executive Director J.D. Froula reported on the status of the 2011 Alumnus Giving Program, the AGP Multi-State Registration Project, and the first initiation of 20 initiates of Texas Delta at Texas A&M University at Qatar on April 20. The Council amended and approved the 2011-12 budget and a plan for the July 31 allocation transfers to the trust funds. A bequest of \$5,000 was received from the estate of E.G. Lucken, *NY I '42*, and two named TBPI-Lucken Scholarships will be awarded.

The Council established the TBPI Audit Committee and appointed Councillors Norman Pih and Jason A. Huggins, P.E., to the committee.