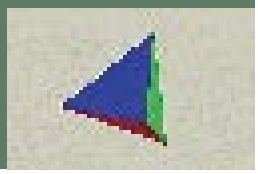


Brain Ticklers

RESULTS FROM SPRING 2010

Perfect

*Chatcavage, Edward F.	PA	B '80
Couillard, J. Gregory	IL	A '89
Fenstermacher, T. Edward	MD	B '80
Gay, Nicholas J.	KS	Γ '09
Jones, John F.	WI	A '59
*Kimsey, David B.	AL	A '71
*Mangis, J. Kevin	VA	A '86
*Marx, Kenneth D.	OR	A '61
Prince, Lawrence R.	CT	B '91
Rasbold, J. Charles	OH	A '83
*Schmidt, V. Hugo	WA	B '51
Silver, Robert E.	NY	P '80
*Spong, Robert N.	UT	A '58
Stribling, Jeffrey R.	CA	A '92
*Strong, Michael D.	PA	A '84
Stubler, William G.	NY	N '07
Winn, Andrew K.	NY	Γ '09

Other

Alexander, Jay A.	IL	Γ '86
Aron, Gert	IA	B '58
Bachmann, David E.	MO	B '72
Bentley, Brook I.	OH	H '09
Bertrand, Richard M.	WI	B '73
Brown III, William T.	NC	E '89
Coomer, Maxwell R.	TN	A '95
Egenriether, Brian J.	SC	Γ '10
Handley, Vernon K.	GA	A '86
Harris, Kent	Non-member	
Hess, Richard I.	CA	B '62
James, Catherine A.	Wife of member	
Jenneman, Jeffrey H.	OK	A '08
Jones, Donlan F.	CA	Z '52
Kaatz, Leon M.	Cousin of member	
Kaliski, Burton S., Jr.	MA	B '84
Kaliski, Michele F.	NY	Δ '85
Kaliski, Stephen	Son of members	
Lynch August, Jacob D.	WI	Δ '09
Marrone, James I.	IN	A '61
Meixner, David P.	IL	A '06
Norris, Thomas G.	OK	A '56
Oar, Tyler D.	CO	Z '11
Oliver, Christopher R.	AL	E '08
Pineault, Wayne	IL	Γ '79
Pittard, Mathew T.	UT	B '01
*Rentz, Peter E.	IN	A '55
Bamford, Bob	Non-member	
Sigillito, Vincent G.	MD	B '58
Summerfield, Steven L.	MO	Γ '85
Sutor, David	Son of member	
Voellinger, Edward J.	Non-member	
*White Jr., Warren N.	LA	B '74

* Denotes correct bonus solution

In Memory

We sadly note the passing of Martin Gardner, who died on May 22, 2010, at the age of 95. Gardner was perhaps the finest recreational mathematician of our time and a prolific author with more than 70 books to his credit. He will be greatly missed.

SPRING REVIEW

The most difficult regular problem was No. 3, about the gambling game Lucky-10. The bonus about the hula-hoop was fairly difficult with only 20% of entries having a correct answer.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter '11 BENT. Meanwhile, here are the answers:

1 The only prime of the form A^4+4B^4 is 5. To see this, note that A^4+4B^4 can be factored as $(A^2 + 2AB +2B^2) (A^2 - 2AB +2B^2)$. One of these factors must be one, the other prime. Only the second factor can equal one when A and B are positive integers, and $A=B=1$ is the only solution. The other factor yields 5.

2 Clarence killed Popoff. Abbreviate the numbered statements made by Algernon (A), Bertie (B), or Clarence (C) as true (T) or false (F). For example, A2=F means that Algernon's 2nd statement is false. It is easy to see that the number of T statements made by A and B cannot be either 0 or 3 as this quickly leads to a contradiction. So consider the number of true statements made by A and B to be 2. Then, A2=F is the only possible false statement of A so A1=T and A3=T. A3=T implies C1=T. But C1=T contradicts A1=T. Thus, the number of true statements made by A and B must be 1. A2=F must be one of the two false statements of A. First consider A1=F and A3=T. This implies C1=T, C2=T, and C3=T. C2=T implies B1=F. Also, we must have B2=F if one of A's statements are T. This leads to B3=T, a contradiction with C's statements being T. Thus, we must have A1=T, A2=F, and A3=F. There are two possibilities for B's T statement; first consider B1=F, B2=F, and B3=T. B1=F implies C2=T. But this contradicts B3=T. Thus, we must have B1=T, B2=F, and B3=F; this implies C1=F, C2=F and C3=T. This means that A did not see Popoff before the murder and B never handled a dagger. Thus, C murdered Popoff.

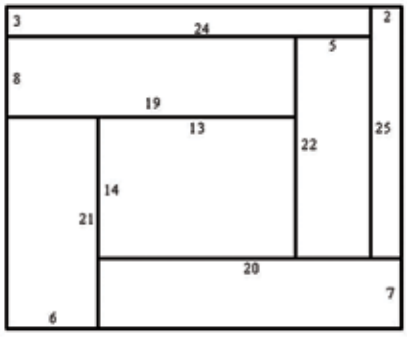
3 There are 10 class II gunboats in the Patagonian navy. Let N_I, N_{II}, and N_{III} equal the number of class I, II, and III gunboats, respectively. Each flotilla has 6 ships and at least one of each class, so the minimum number of guns in a flotilla is 21; the maximum is 27. Since each of the 7 flotillas has a different number of guns, these numbers are 21, 22, 23, 24, 25, 26, and 27, a total of 168 guns. Now, the total number of class II and III ships is $N_{II} + N_{III} = 6(7) - 16 = 26$, and the total number of guns on these ships is $4N_{II} + 3N_{III} = 168 - 5(16) = 88$. Solving these equations yields $N_{II} = 10$ and $N_{III} = 16$.

4 The release point of the mail occurs at x=0.20558 when the plane is flying in the $+x$ direction and at x=1.42422 when flying in the $-x$ direction. The equations in time t for the trajectory of an object falling under the influence of gravity are $z(t) = z_r + v_z t + at^2/2$ and $x(t) = v_x t + x_r$, where (v_x, v_z) are the (x, z) components of the release velocity and (x_r, z_r) is the release point. Keeping in mind that $v^2 = v_x^2 + v_y^2 = 1$ and $a = -1$ gives $z(t) = -½ t^2 + v_z t + z_r$ and $x(t) = v_x t + x_r$. From the plane's flight path, $z=x^3$, we find the initial angle of fall as $dz/dx = \tan\theta = 2x$. Thus, $v_x = v\cos\theta = 1/(1+\tan^2\theta)^{½} = 1/(1+4x^2)^{½}$ and $v_z = v\sin\theta = \tan\theta/(1+\tan^2\theta)^{½} = 2x/(1+4x^2)^{½}$. Since the final value of x is 1, the equation for $x(t)$ becomes $1 = t\cos\theta + x_r$. Solving for t yields $t = (1-x_r)/\cos\theta = (1-x_r)(1+4x_r^2)^{½}$. Substituting this value of t into $z(t)$ yields $-½(1-x_r)^2(1+4x_r^2) + 2x_r(1-x_r)+x_r^2 = 0$, as $z = 0$ at the mail's receipt point. This equation reduces to $4x^4-8x^3+7x^2-6x+1 = 0$ and yields real roots at x = 0.20558 and 1.42422.

5 Ignoring the energy used by the belt sander when not sanding, 11.3 watts are dissipated to friction when the pin's rotation is opposite that of the belt and 8.69 watts when rotating in the same direction. The unit of power is the watt = 1 Joule/sec, so consider the energy dissipated per second. Joules = Newtons x meters, so calculate the distance over which the horizontal force is applied. In one second, the pin rotates 100/60 revolutions, and for each revolution the pin's surface travels a

distance of $\pi D = 0.05\pi = 0.157$ meters for a total of $0.157(100/60) = 0.262$ m/s. When the pin is rolling opposite to the belt sander's direction, the relative distance traveled in one second is $2 + 0.262 = 2.262$ m. The perpendicular force applied is 10 Newtons, and the coefficient of friction is 0.5. The power dissipated due to friction is then $0.5(10)(2.262) = 11.3$ watts. When the pin is rotating in the same direction as the sander, the relative distance traveled at the pin's surface is $2\text{-}0.262 = 1.738$ meters, and the power dissipated is $0.5(10)(1.738) = 8.69$ watts.

Bonus. The dimensions of George's patio are 26x32; the internal rectangles have dimensions (24,3), (2,25), (20,7), (6,21), (19,8), (5,22), and (13,14). Visualize a large rectangle with 7 interior rectangles, a different rectangle at each corner, and five different rectangles around the perimeter, similar to the figure below. Let the dimensions of the internal rectangles be (x_i, y_i), i=1,...,7. Eight equations can be written by taking horizontal (x) and vertical (y) cuts through the large rectangle. One choice yields $A=x_1+x_2=x_3+x_4=x_2+x_5+x_6=x_2+x_4+x_6+x_7$ and $B=y_2+y_3=y_1+y_4+y_5=y_1+y_3+y_6=y_1+y_3+y_5+y_7$. Use the semi-perimeter equations $y_i=S-x_i$ to express the y_i in terms of the x_i, reducing the problem to 8 equations in 10 unknowns (seven x_i's, A, B, and S). Use 6 of these equations to express x_2 through x_7 as functions of x_1, A, B, and S ($x_2=A-x_1$; $x_3=2S-A-B+x_1$; $x_4=2A-2S+B-x_1$; $x_5=5S-2A-2B$; $x_6=2A+2B-5S+x_1$; $x_7=7S-4A-3B+x_1$). Substituting these values into the remaining 2 equations gives $B=3S-x_1-x_3-x_6$ and $B=4S-x_1-x_4-x_5-x_7$. Rearranging gives $3x_1=6S-A-2B$ and $3x_1=7A+5B-10S$. Adding these equations gives $4S=6A+3B-6x_1$, showing that S is divisible by 3, and equating them gives $7B=16S-8A$, showing that B is divisible by 8. A little thought indicates that S_{min}=15, and since S<30, S=15, 18, 21, 24, or 27. Further thought indicates that $8<B<53$; so B=16, 24, 32, 40, or 48, and there are only 25 combinations of S and B to try (easily done on a spreadsheet). Pick S and B, calculate A and x_1, and then calculate all the other x_i and y_i to see if this gives 14 all different positive integers for the x_i and y_i. Only S=27 and B=32 gives a solution (see figure).



Double Bonus. To maximize the area of an N-sided polygon, arrange the N sides so that all the vertices are on a circle, the figure is convex, and no sides cross (the order of the sides does not matter). This is known as the area theorem of a cyclic polygon. Proof: (a) If any two sides cross, a reflection of the sides between the vertices of the two sides that cross will increase the area. (b) If any part of the figure is concave, a reflection of the concave part into its convex mirror image will increase the area. (c) Form a triangle from two adjacent sides by joining their un-joined ends. You can swap the order of the two sides without altering the area of the triangle, and, hence, without altering the area of the polygon. Thus, the order of the sides does not matter. (d) Arrange the vertices of the polygon to lie on a circle, forming a circle with N segments. Assume that the vertices of the polygon act like pivots, allowing the figure to be reshaped without changing the length of its perimeter. Since a circle maximizes the enclosed area, reshaping will reduce the area enclosed and, since the area of the segments remains constant, reduce the area of the polygon. Hence, the maximum area occurs when the vertices lie on a circle. A good reference is: *www.mast.queensu.ca/~peter/investigations/24polygon_area.pdf.*

NEW FALL PROBLEMS

1 Ann, Barb, and Cleo are three remarkable women: two are remarkably intelligent, two are remarkably beautiful, two are remarkably artistic, and two are remarkably rich. Each has no more than three remarkable characteristics. If Ann is remarkably intelligent, she is remarkably rich. If Barb or Cleo is remarkably beautiful, she is remarkably artistic. If Ann or Cleo is remarkably rich, she is remarkably artistic. Which woman is not remarkably rich?

—*Test Your Logic* by George J. Summers

2 A researcher is interested in studying cheating at his *alma mater*, but he feels that if he asks students directly if they cheat, he will not get an honest answer. Therefore, he decides to use the following procedure. He places three cards in an opaque bag. The first card reads, "Have you cheated on a test in the past year?"; the second card reads, "Answer Yes," and the third card reads, "Answer No." He hands the bag to a student, asks him to select a card at random from the bag, answer Yes or No, and replace the card in the bag, all without letting the researcher see the card. The researcher interviewed 2,352 students and received 928 yeses and 1,424 nos. Assuming that all of the students either answer truthfully or answer yes or no when directed, what is the estimated percentage of students who cheat?

—*Why Do Buses Come in Threes?* by Rob Eastaway & Jeremy Wyndham

3 A banker agrees to lend you a hundred million dollars at an interest rate of 100%. If the compounding period for the interest is yearly, then you will owe the banker two hundred million dollars after one year (assuming that you make no other payments). However, if the interest rate is compounded monthly at an interest rate of 1/12 of 100%, then you will owe $261,303,529.02 after one year; if the interest is compounded weekly at a rate of 1/52 of 100%, you will owe $269,259,695.44. How much will you owe, to the nearest penny, after one year, if the interest is compounded continuously?

—**John L. Bradshaw,** *PA A '82*

4 Three spheres of radii 2, 3, and 4 cm lie on a horizontal plane, tangent to each other. A fourth sphere of radius 1 cm is placed in the pocket formed by the other three. What is the distance of the center of the smallest sphere from the plane surface?

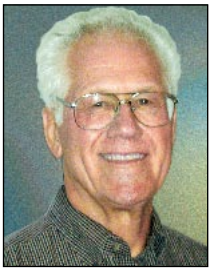
—*Technology Review*

(Continued on page 36.)

LYLE'S LAW

(Continued from page 16.)

before you go, but you will still have a lot to learn once you get there. Probably your best ally in your early days abroad will be a heightened sense of humility. That and a few words of the local language will go a long way to keeping you comfortable while you learn the local customs. A few years ago, my wife and I were in a little restaurant in Perpignan, France. I used my few words of atrocious French to apologize for my inability to speak their language and found that the waitress spoke reasonably good English. Across the room was a trio of Americans who not only didn't try to speak a little French, they made fun of the language. Hey, I think I'll order the poison. Ho. Ho. Ho. Funny, but that waitress forgot every word of English whenever she crossed the room. Learn your territory, folks.



Now, learning the territory doesn't necessarily mean that you adapt completely to the culture, be it that of a company, a country, or a club. If the inhabitants of the territory behave in a way that offends your ethical standards, by no means should you conform. There may or may not be opportunities to change the culture, but, if not, unto thine own self be true. In any event, first you gotta—oh, please—you have to know your territory.

125TH ANNIVERSARY CLUB

2010 celebrates the 125th anniversary of the founding of Tau Beta Pi. To commemorate this historic occasion, a one-time recognition club has been established for donors contributing $125 or more in 2010. (Members of existing clubs will still be listed in those clubs.) Donors of $125 or more will receive a striking memento acknowledging their special support during the Quasquicentennial celebration. This limited-edition item features the Association's 125th anniversary logo etched onto a polished 3" x 3" black marble paperweight. For more information, contact Patricia McDaniel—*pat@tbp.org*, 865/546-4578.

BRAIN TICKLERS

(Continued from page 35.)

5 A young girl is riding in a swing with chains 8 ft in length and a seat 2 ft above the ground. She wants to jump off and land as far from the swing's rest position as possible. At what angle of the swing (the angle is 0° when the chains are vertical) should she release from the seat, and how far from the point directly under the swing's rest position can she land? Treat the girl as a point mass, and assume that the highest she can pump the swing is 45° with respect to vertical.

—Towing Icebergs, Falling Dominoes, and Other Adventures in Applied Mathematics
by Robert B. Banks

Bonus. A football punter, standing at the middle of the 50-yard line, wishes to make a "coffin corner" kick. That is, to kick the ball out of bounds as close to the goal line as possible, but not into the end zone. If successful, the opposing team will get the ball at the yard line where it goes out of bounds; if unsuccessful, it will get the ball at the 20-yard line. Assume the ball travels in a straight line, has enough velocity to cross the sideline or goal line, and there is no run-back. If the kicker's launch angle has a normal distribution with a standard deviation of 7.5° from where he aims, at what sideline yard marker should the punter aim so that the expected value of the opposing team's starting position is minimized? What is that expected value (in yards from their goal line)? A football field, not counting the end zones, is 300 feet by 160 feet.

—Steven E. Bradley, *AZ A '77*

Computer Bonus. The smallest palindromic square with an even number of digits is 698,896 (the square of 836). What are the next two smallest palindromic squares with an even number of digits?

—The Colossal Book of Mathematics by Martin Gardner

Send your answers to any or all of the Fall Brain Ticklers to **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697**, or email plain text only to *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT in December. The method of solution is not necessary. We also welcome any interesting problems that might be suitable for use in the column. The Computer Bonus is not graded. Jim will forward your entries to the judges who are **H.G. McIlvried III,** *PA Γ '53;* **F.J. Tydeman,** *CA Δ '73;* **D.A. Dechman,** *TX A '57;* and the columnist for this issue,

—J.L. Bradshaw, *PA A '82.*