Brain Ticklers

RESULTS FROM SPRING 2009

Perfect			
*Couillard, J. Gregory	IL	Α	'89
*Fenstermacher, T. Edward	MD	Β	'80
Schmidt, V. Hugo	WA	Β	'51
*Spong, Robert N.	UT	Α	'58
*Venema, Todd M.	OH	Η	'08
Other			
Alagappan, Vairavan	IL	Α	'86
Beans, E. William	OH	Γ	'53
Bertrand, Richard M.	WI	Β	'73
Bohdan, Timothy E.	IN	Γ	'85
Brule, John D.	MI	Β	'49
*Doniger, Kenneth J.	CA	Α	'77
Harris, Kent	Non-member		
James, Catherine A.	Wife of member		
*Kimsey, David B.	AL	Α	'71
Lew, Thomas M.	TX	Δ	'84
*Midgley, James E.	MI	Γ	'56
*Newmiller, Jeffrey D.	CA	Λ	'86
*Quintana, Juan S.	OH	Θ	'62
*Rasbold, J. Charles	OH	Α	'83
Rogaski, C.	Parent of member		
Routh, Andre G.	FL	Β	'89
Silver, Robert E.	NY	Ρ	'80
*Stribling, Jeffrey R.	CA	Α	'92
*Strong, Michael D.	PA	Α	'84
Sutor, David	Son of member		
Voellinger, Edward J.	Non-member		
Washburn, Alan R.	PA	Γ	'62
*Weinstein, Stephen A.	NY	Γ	'96

* Denotes correct bonus solution

SPRING REVIEW

The spring set seems to have been a little harder than usual, with only a few perfect scores. The easiest problem was No. 3, the TBPI cryptic, and the hardest was No. 5 about the poker hand, which proved to be harder than the Bonus.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter 2010 Bent. Meanwhile, here are the answers:

1 The problem asks whom Mrs. Zenith does not invite to her hosted bridge game; the answer is Mrs. Virtue. Since no woman invites herself, we have Vxyz (meaning V invites x, y, and z) and Xw--, Yw--, and Zw--. Now, in addition to w, X must invite two of vyz. If Xvwy with z free, then Wvy- with the fourth player being x or z, but it can't be x because that would give z two free days, so Wvyz with x free. Now, Y must invite two of vxz, but it can't be vz because this would give x two free days, and it can't be vx because this would give z two free days. Therefore, Ywxz, but this violates the condition that W, as her fourth, invited a lady not among those invited by Y. Similar reasoning shows that X cannot invite yz. Therefore, Xvwz, Wvyz, Yvwx, and Zwxy, which means V does not get an invitation from Z. The tricky part of this solution is that W invites Y as her fourth, but since Y did not invite herself, the invitation to Y does not violate any conditions of the problem.

2 The problem asks for the probability that three sharpshooters hit the same hemisphere of a rapidly spinning sphere under two different cases. For Case (1), the probability that all three shots are in the same hemisphere is 100%; for Case (2) it is 75%. For Case (1), draw a great circle through the first two hit points; then both shots are for all practical purposes in both hemispheres. Now, the third shot must be in one or the other hemisphere. Therefore, the probability that all three shots are in the same hemisphere is 100%. For Case (2), pass a plane through the first bullet hole and the axis of the sphere, and pass a second plane through the second bullet hole and the axis. Let the angle between these two planes be Θ. Then, if the third bullet lands in the lune (formed by the two planes and the sphere) opposite the lune containing the first two bullet holes, it will not be in the same hemisphere as the other two shots. This lune occupies a fraction of the sphere's area equal to $\theta/2\pi$. Therefore, the probability that the third shot is not in the same hemisphere is the integral from 0 to π of $(\theta/2\pi)\, d\theta/\pi = ¼$. Therefore, the probability that the third shot is in the same hemisphere is ¾.

3 The solution to SEVEN + THREE + TWO = TWELVE is 82524 + 19722 + 106 = 102352. A five-digit number (SEVEN) plus a five-digit number (THREE) yielding a six-digit number (TWELVE) means T is 1. Since S + T + carry = 10 + W and carry must be 1, then S is 8 and W is 0. Since TWO lies between 102 and 109 and is the product of two primes, TWO must be 106 = 2x53, and O is 6. Since O + N must equal 10, N is 4. E + H + carry = 10 + E, so H + carry = 10 with carry = 1, H=9. Then, E is 2, 3, 5, or 7, but E cannot be 5 as this would make V = 1. Trying E = 3 and E = 7 leads to inconsistencies, but E = 2 gives the above answer.

4 The shortest possible marble and saucer game has 12 marbles in the five saucers distributed as follows: Saucer A: (1, 2, 12), Saucer B: (4, 6, 11), Saucer C: (7, 10), Saucer D: (5, 9), Saucer E: (3, 8). The solution to this problem involves the application of two conditions. Condition one is to have marbles with a difference of 1 in Saucer A; marbles with a difference of 2 in B; a difference of 3 in C; etc. Condition two is to have marbles with consecutive numbers in the five saucers with the highest number in A. If these conditions are met, then the next marble cannot be placed in any of the saucers. Neither 10 nor 11 marbles allow these conditions to be met, but 12 marbles work when distributed as shown above. It is fairly easy to find this solution if you start by placing marbles 8 to 12 first. Then 7 goes in C, 5 in D, and 3 in E. This leaves 1 and 2 for A and 4 and 6 for B.

5 The value of: $\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(2...)})})})})}$ is 2. Let $x = \sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(2...)})})})})}$. Then, $x^2 = 2\sqrt{(2\sqrt{(2\sqrt{(2\sqrt{(2...)})})})} = 2x$. Therefore, x = 2.

Bonus. The required number of locks on the safe so that only a majority of the owners can open it is C(N, k), where k = (N-1)/2 and C(i, j) is the number of combinations of i things taken j at a time. Thus, C(N, k) is the number of ways to pick a simple majority of k +1 members from the N owners. The approach is to add one lock for each different simple majority and hand out k+1 keys, one to each member of that majority. Thus, the total number of locks is L = C(N, k) = N!/[(N+1)/2]![(N-1)/2]!, and the total number of keys is $L(k + 1)$. Each owner gets $L(k + 1)/N = L(k + 1)/(2k + 1) = (2k)!/(k!)^2$ keys. Some values for L are: N = 3, L = C(3, 2) = 3; N = 5, L = C(5, 3) = 10; N = 7, L = C(7, 4) = 35; and N = 9, L = C(9, 5) = 126. The following table shows a distribution of keys for five owners (1-5) and 10 locks (A-J). Similar tables can be easily prepared for other cases.

	A	B	C	D	E	F	G	H	I	J
1	X	X	X	X	X	X				
2	X	X	X				X	X	X	
3	X			X	X		X	X		X
4		X		X		X	X		X	X
5			X		X	X		X	X	X

Double Bonus. The American hero is Benjamin Franklin. 2006 was the 300th anniversary of Franklin's birth. Let B be the year of his birth, D be the year of his death, X be the years before 2006 that he was born, and Y be his age at death. Therefore, X = 2006 – B and $Y = D - B$. Let Y = 10M + N; then Y minus its inverse equals $10M + N - 10N - M = 9(M - N) = (X - Y)^{2/3}$, or $27(M - N)^{3/2} = X - Y$. Therefore, $M - N$ is a perfect square (1, 4, or 9). Now, X = 10M + N + 27(M – N)$^{3/2}$. Since B = 2006 – X is twice a prime, X must be even. Trying all combinations of $M = N + 1$, $M = N + 4$, and $M = N + 9$ that give an even X, we get as possible values for X: 48, 70, 92, 114, 256, 278, and 300. Of these, only 300 (with M = 8, N = 4, and X = 10M + N = 84) gives a value for B that is twice a prime (B = 2006 – X = 2006 – 300 = 1706 = 2x853). Therefore, the hero was born in 1706, died in 1790 at age 84, and 2006 was the 300th anniversary of his birth.

NEW FALL PROBLEMS

1 In cross-country matches, teams of six runners compete against each other. Team scores are determined by adding the finishing positions of the first four runners to cross the finish line on each team. The lowest score wins. The fifth and sixth runners on each team do not score, but they do count in determining the positions of runners following them. Individuals never tie for any position. If two teams have the same score, the team whose fourth runner has the lower score wins. In a recent two-team match, Al was a non-scorer on the winning team. Each team's score was a prime number, and if you knew those scores, you could deduce the finishing positions of all the runners on each team. You don't know those scores, but if you knew Al's position, together with the above information, you could again deduce all the finishing positions. What was Al's finishing position and those of the scorers on his team?

—Richard England in *New Scientist*

2 In the following cryptic addition: ALCOHOL + ALCOHOL + ... + ALCOHOL = HANGOVER , how much ALCOHOL is needed? There are no leading zeros, different letters are different digits, and same letter is same digit throughout.

—recreational puzzles newsgroup

3 Consider a circle with a unit radius. Inscribe an equilateral triangle in this circle. Then, inscribe another circle in the triangle. Next, inscribe a square in this circle and another circle in the square, and then inscribe a regular pentagon. Continue this process, each time inscribing a circle followed by a regular polygon with one more side. What is the radius of the limiting circle?

—*Mathematical Sorcery* by Calvin C. Clawson

4 Three points are chosen at random in a hemisphere of radius R. What is the expected value of the distance of the closest point from the hemisphere's flat surface?

—**Russell T. Nevins,** *MA B '77*

5 Let N be a positive integer, and let S(N) represent the sum of the digits of N. For example, S(1249) = 16, and S(12^2) = 9. What is the value of S(S(S(4444^{4444})))?

—*Classic Mathemagic* by R. Blum, A. Hart-Davis, B. Longe, and D. Niederman

Bonus. A one-meter massless string is attached to the end of a vertically rotating shaft, with a mass of 25 g attached to the other end of the string. The rate of rotation is such that the string makes an angle of α with the vertical. The mass is then given a small outward horizontal impulse that causes it to oscillate up and down across the circular path it had been following. Find an expression for the frequency of oscillation.

—John R. Sellars

Double Bonus. Given that S(n) = 1(1!) + 2(2!) + 3(3!) + 4(4!) + … + n(n!), what is the value of S(n)?

—*The Mathematics Teacher*

Postal mail your answers to any or all of the Brain Ticklers to: Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697 or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT around early January. The method of solution is not necessary.

We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53;* **F.J. Tydeman,** *CA Δ '73;* **D.A. Dechman,** *TX A '57;* and the columnist for this issue,

—**J.L. Bradshaw,** *PA A '82*