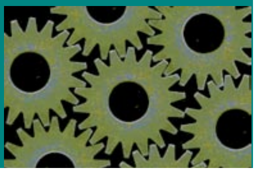

Brain Ticklers

RESULTS FROM SPRING 2008

Perfect

	Ballard, Jeffrey A.	FL E '05
*	Bohdan, Timothy E.	IN Γ '85
*#	Celestino, James R.	NJ B '00
	Couillard, J. Gregory	IL A '89
*	Fenstermacher, T. Edward	MD B '80
*	Foster, David E.	VA B '94
*#	Gerken, Gary M.	Non-member
*	Kimsey, David B.	AL A '71
*	Prince, Lawrence R.	CT B '91
*#	Rasbold, J. Charles	OH A '83
*	Schmidt, V. Hugo	WA B '51
*	Strong, Michael D.	PA A '84
*	Thaller, David B.	MA B '93
	Treacy, Stephen V.	NY O '89
#	Voellinger, Edward J.	Non-member
	Wolff, Nicholas L.	NE A '00
#	York, Jeffrey A.	NC A '85

Other

	Alexander, Jay A.	IL Γ '86
	Aron, Gert	IA B '58
	Bachmann, David E.	MO B '72
*	Berger, Toby	CT A '62
	Brule, John D.	MI B '49
#	Delang, William	WA A '61
	deVitry, David M.	PA H '97
	Doniger, Kenneth J.	CA A '77
*	Forest, Thomas M.	MI Z '87
	Harris, Kent	Non-member
	Jones, Donlan F.	CA Z '52
	Marrone, James D.	IN A '87
	Mazeika, Daniel F.	PA B '55
	Nabutovsky, Joseph	Father member
	O'Reilly, Thomas B.	VA B '96
	Oliver, Christopher R.	AL E '08
	Quintana, Juan S.	OH Θ '62
	Rentz, Peter E.	IN A '55
	Scholz, Gregory R.	PA B '00
	Silver, Robert E.	NY P '80
	Skowronski, Victor J.	NJ A '71
#	Spong, Robert N.	UT A '58
*#	Stribling, Jeffrey R.	CA A '92
	Sutor, David	Son of member
	Takahashi, Tsuyoshi	Non-member
	Tessier, Thomas M.	MA A '90
	Weinstein, Stephen A.	NY Γ '96

* Denotes correct bonus solution
# Denotes correct computer bonus solution

SPRING REVIEW

The regular spring problems turned out to be about equally difficult, except for No. 5 about the sum of the inverses of the triangular numbers, for which over 80 percent of the entries had correct answers. K.J. Doniger, *CA A '77*, and D.E. Foster, *VA B '94*, provided recursive formulas to solve the Bonus problem.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter Bent. Meanwhile, here are the answers:

1 The successful pairs for the photography safari were: P and Q photographed the antelope, P and T photographed the baboon, R and S photographed the crocodile, S and T photographed the dromedary, and Q and R photographed the elephant. Since there are five people, there are 10 possible pairs, and each person went out four times, each time with a different partner, and twice got a photograph and twice got nothing. First determine which pair photographed something and which didn't by making a 5x5 matrix with row and column headings of P, Q, R, S, and T. Assign the animals numbers from 1 to 5 and determine which numeral is which animal later. Since (P, Q) got something, assign (P, Q) = (Q, P)=1 in the matrix. Since everybody got two animals, you can assign animal 2 to P [(P, RST) = 2], and animal 3 to Q [(Q, RST) = 3] in the matrix. This leaves the assignment of animal 4 to S and animal 5 to T [(R, S) = 4 and (S, T) = 5]. Now, since S has both animals 4 and 5, S cannot have animals 2 and 3, which means that we can assign 2 to R and 3 to T. The successful pairs are then (P, Q), (P, T), (Q, R), (R, S), and (S, T). Next, determine which pair photographed which animal. We know that E is 1 or 3, A is not 2 and 5, B is not 4 and 5, and B is therefore 2 or 3. Consider the cases (E, B) = (1, 2), (1, 3) and (3, 2) separately. The first two cases lead to contradictions; (E, B) = (3, 2) leads to C=4. Then, since T did not photograph A, A cannot be 2 or 5, so A must be 1. Then D is 5.

2 The set of three different integers, P, Q, and R, such that P+Q, P+R, Q+R, P-Q, P-R, and Q-R are all squares of integers and P+Q+R is minimized, is {P,Q,R} = {17, 8, -8}. Taking sums and differences of the equations involving P and Q reveals that 2P is the sum of two squares in two different ways, 2R is the difference of two squares in two different ways, and 2Q is both the sum of two squares and the difference of two squares. Therefore, the squared integers in the equations for 2P, 2Q, and 2R must be either both even or both odd. Since we want the minimum solution for P+Q+R, consider cases in which the integers that are squared are small, say zero through ten. Working through these trials then yields the above solution.

3 The first hand bet 60 cents in the stud poker game. Since there are six denominations of U.S. coins, there are only 21 possible bets that the first hand can make with two coins. Of these, only 12 bets can be matched with a three-coin bet in the second hand. For the third and fourth hands, start considering cases to eliminate bets that can't work; determine the sum of the possible coinage put in from the number of coins removed; and then determine if that amount can be bet using two coins (third hand) or three coins (fourth hand). The fifth hand then determines the single answer.

4 The integral solutions of $(x+1)^y = x^{y+1} + 1$ are: $(x, y) = (0, [0,1,2,3,\ldots])$, (1, 1), (-1, [2,4,6,8,…]), (2,2). The first and second solutions are found by looking at those cases in which the right-hand side of the equation is one or two, respectively. The third solution is found by looking at the case in which the right-hand side of the equation is zero. (2, 2) is a little harder but can be found when considering the cases for which the right-hand side of the equation is a perfect square, cube, etc.

5 The probability that four people flip a fair coin five times and get the same number of heads is 5,313 /262,144 or approximately 0.02067. When you flip a fair coin five times, the probability of getting heads n times is $p(n) = C(5, n)(1/2)^5 = 5!/[n!(5 - n)!](1/2)^5$. The probability of four people flipping n heads in five tries is $[p(n)]^4$, and the probability of four people flipping the same number of heads in five tries is $\Sigma_0^5 [p(n)]^4$.

Bonus. The problem asked for two equilibrium positions of five identical charges free to move on a unit square. There are actually many solutions; anyone submitting two or more positions got full credit. To take full advantage of symmetry, consider a symmetrical solution with one charge at the origin (0, 0) and the other charges at (0.5, y), (-0.5, y), (x, 1), (-x, 1). Note that sum of the forces on the charge at the origin in the direction along the wire is identically zero for any choice of x and y. The forces on (0.5, y) come mainly from (0, 0) and (x, 1). The force from (x, 1) in the vertical direction is stronger because the vertical force from (0, 0) includes the cosine of the horizontal angle between the charges at (0, 0) and (0.5, y), and this weakens the force more than the similar cosine effect from the charge at (x, 1). Thus the forces on the charge at (0.5, y) will push it toward the corner at (0.5, 0). Now there are three beads pushing (x, 1) to the right, while only (0.5, 0) is pushing it to the left; so x goes to 0.5, and the five beads end up at (0, 0), (0.5, 0), (-.5, 0), (0.5, 1), and (-.5, 1), or a midpoint and four corners. For another symmetrical solution, consider charges at (0, 0), (w, 0), (0, w), (z, 1), and (1, z). Note that the forces on the charge at the origin keep it in the corner for all possible w and z. For w near 0.5, consideration of the forces on the charges at (w, 0) and (0, w) indicates that these charges will be forced to the corners at (1, 0) and (0, 1). So with three charges at (0, 0), (1, 0), and (0, 1), write down the vertical force (along the wire) on the charge at (1, z) as $f_1 + f_2 + f_3 + f_4 = 0$, where $f_1 = 1/z^2$, $f_2 = z/(1+z^2)^{3/2}$, $f_3 = -(1-z)/(1+(1-z)^2)^{3/2}$, and $f_4 = -1/2^{3/2}/(1-z)^2$. The value of z for which the total vertical force is zero is $z = 0.630624$; so a second solution has charges at (x, y) = (0, 0), (1, 0), (0, 1), (1, 0.630624) and (0.630624, 1). Both of the above solutions are stable, that is, small displacements of any charge from its equilibrium position results in forces that push all charges back toward their equilibrium positions. Two additional stable solutions have charges at [(0,0), (1,0), (0,1), (0.323835,0), (0.685923,0)] and [(0, 0), (1, 0), (0, 1), (0.547530, 0), (0, 0.547530)]. We also found eight unstable solutions.

Computer Bonus. The sum of the factors, (including 1 and the cube) of the cube of 7 is a perfect square. The next largest such number having this property is 751,530. Note: $751{,}530^3 = 424{,}462{,}145{,}606{,}577{,}000 = 2^3 3^3 5^3 13^3 41^3 47^3$. The sum of the factors of 424,462,145,606,577,000 is 1,669,404,572,559,360,000, which equals $1{,}292{,}054{,}400^2$. If N = $p_1^a p_2^b p_3^c \ldots$, then the sum of the factors of N is $S = [(p_1^{a+1} - 1)/(p_1 - 1)][(p_2^{b+1} - 1)/(p_2 - 1)][(p_3^{c+1} - 1)/(p_3 - 1)]\ldots$.

NEW FALL PROBLEMS

1 When the five marriage counselors at Healthy Relations got divorced, it was a shock. When all announced remarriage and the brides were revealed to be the five ex-wives, it was a sensation. Still, the priggish Dinah was not the first to remarry and there were no direct swaps, so the decencies were preserved. The weddings were held on successive Saturdays. Peter's took place earlier than Anne's and later than Quentin's. Barbara's was later than Tristram's and earlier than Celia's. Peter married Simon's ex-wife. Barbara got hitched to the man whose former wife married Emily's ex-husband. Quentin paired up with the lady whose former husband married Dinah. Ronald was spliced with the lady whose ex-husband married Celia. Who remarried whom? No one remarried his or her ex-spouse.

—*Martin Hollis*

2 Place eight queens on a standard chessboard so that the number of squares not attacked is a maximum. What is this number? Describe the positions of the queens, labeling columns *a-h* and rows *1-8*.

—*Technology Review*

3 The result of adding the date of the last Monday of last month and the date of the first Thursday of next month is 38. If both dates are in the same year, what is the current month?

—*Hard-to-Solve Brainteasers* by Jamie and Lea Poniachik

4 Starting with a standard 52-card deck and drawing cards randomly without replacement, what is the expected number of cards that must be drawn until each of the four suits is represented at least once?

—*The Theory of Gambling and Statistical Logic* by Richard A. Epstein

5 A stick of length L is standing vertically against a wall, with a small beetle sitting on top of the stick. The stick slides down the wall at a constant rate in such a way that the top maintains contact with the wall and the bottom maintains contact with the floor.

The beetle starts to crawl down the stick at a constant speed just as the stick starts to slide. If the beetle reaches the bottom of the stick just as the top of the stick reaches the floor, provide the equation of the beetle's path in terms of Cartesian coordinates only.

—*Jeffrey R. Stribling, CA A '92*

Bonus. A spherical bead of mass M is free to slide along a frictionless, horizontal wire. Hanging from this bead at the end of a massless wire of length L is a lighter sphere of mass m. If the lighter sphere is given a small displacement in a direction parallel to the horizontal wire, what is the period of the resulting oscillation?

—*John R. Sellars*

Double Bonus. The infinite horn of plenty for $x \geq 1$ is generated by revolving the curve $y = 1/x$ about the x-axis. (For this problem, let x and y be in feet.) Your job is to paint the inner surface of the horn using the new nanotech paint. Nanotech paint comes in one gallon cans, can be applied as an infinitely thin layer and dries that way, but it still takes one gallon of paint for each 10,000 square feet of surface area. How would you paint the inner surface of the horn, and how much paint would it require?

—*John L. Bradshaw, PA A '82*

Postal mail your answers to any or all of the Brain Ticklers to: **Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697,** or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT in late December. The method of solution is not necessary. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53*; **F.J. Tydeman,** *CA Δ '73*; **D.A. Dechman,** *TX A '57*, and the columnist for this issue,

—**J.L. Bradshaw,** *PA A '82*