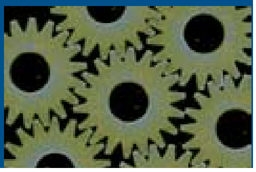


Brain Ticklers

RESULTS FROM SPRING 2007

Perfect

	Bassler, Louis J.	AK	A	'82
	Bohdan, Timothy E.	IN	Γ	'85
*	Christianson, Kent B.	CA	A	'85
	Corey, Patrick J.	WA	A	'58
	Corey, Michael	Son		
	Couillard, J. Gregory	IL	A	'89
*	Garnett III, James M.	MS	A	'65
*	Habeck, William H. B.	MA	B	'87
*	Hall, George	Non-member		
*	Kimsey, David B.	AL	A	'71
*	Lewandowski, Jerold J.	NY	Γ	'92
	Lonz, Gary J.	Non-member		
	Oliver, Christopher R.	AL	E	'08
	Sedlak, Matthew	NY	Γ	'78
	Smolen, Richard	NJ	B	'82
*	Strong, Michael D.	PA	A	'84
*	Thaller, David B.	MA	B	'93
*	Verkuilen, William W.	WI	B	'92
	Widmer, Mark T.	OH	A	'84
	York, Jeffrey A.	NC	A	'85

Other

*	Alexander, Jay A.	IL	Γ	'86
	Aron, Gert	IA	B	'58
	Bachynski, Erin	MI	Γ	'09
	Schloemer, Jeff	Non-member		
	Beal, Timothy A.	MI	A	'84
	Brule, John D.	MI	B	'49
	Conway, David B.	TX	I	'79
	DeFillipo, Lawrence E.	NY	O	'79
*	De Vincentis, Joseph W.	TX	Γ	'93
	deVitry, David M.	PA	E	'97
	Griggs Jr., James L.	OH	A	'56
*	Hagadorn, Hubert W.	PA	E	'59
*	Hess, Richard I.	CA	B	'62
	Hitz, Steven G.	WI	Γ	'76
	Hutchinson, George K.	ME	A	'55
	Jones, Donlan F.	CA	Z	'52
	Jordan, R. Jeffrey	OK	Γ	'00
	Lew, Thomas M.	TX	Δ	'84
	Marrone, James D.	IN	A	'87
	Nabutovsky, Joseph	Father		
	Pecsvaradi, Thomas	PA	Z	'64
	Piergiovanni, Polly R.	KS	Γ	'82
	Piergiovanni, A. J.	Son		
	Quintana, Juan S.	OH	Θ	'62
*	Rasbold, J. Charles	OH	A	'83
	Rentz, Peter E.	IN	A	'55
*	Schmidt, V. Hugo	WA	B	'51
	Scholz, Gregory R.	PA	B	'00
*	Spong, Robert N.	UT	A	'58
	Stanley, Eileen C.	NY	Ξ	'06
	Stribling, Jeffrey R.	CA	A	'92
	Summers, Ken	TX	I	'60
	Summers, Mark T.	TX	I	'84
	Van Houten, Karen J.	ID	A	'76
*	Venema, Todd M.	OH	E	'08
	Vinoski, Stephen B.	TN	Δ	'85
	Voellinger, Edward J.	Non-member		

* indicates correct bonus solution

SPRING REVIEW

The Spring Ticklers proved relatively easy. The hardest regular problem was No. 2 about ranking students. Many respondents failed to note that the poorest student was a boy.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter '08 BENT. Meanwhile, here are the solutions:

1 There are 28 Phew users. If you assume that the first man interviewed uses Grunt, you can determine that there is an even number of men on the bus and that every odd man interviewed uses Grunt. Then you can determine that the tallest and nicest men interviewed must be liars and that the number of Phew users cannot be 19, 24, or 13. This means that the fattest man interviewed is also a liar, so that the man interviewed fifth after him is a truth-saying Grunt-user, and so the number of Phew users is 28. If you assume that the first man interviewed uses Phew, similar reasoning arrives at the same answer.

2 The probability that Sam wins \$50 at the roulette table starting with a \$50 bankroll is 0.0627753. First consider the case where Sam only seeks to win \$1 more or quits when his bankroll is down to \$49. In this case, the probability of getting to \$51 is 18/(18+19). For the case of quitting with either \$52 or \$48, the probability of winning \$2 is $18^2/(18^2 + 19^2)$. One can continue this process and show that the probability of winning \$50 (ending with \$100) is $18^{50}/(18^{50} + 19^{50})$.

3 A typographic error made this cryptic division problem difficult to understand, and grading will be lenient; the judges apologize for the error. The answer is the longhand division of 601,304 by 74 = 8,125 with remainder 54.

4 The problem asks where to place eight marks on a yardstick so that all integral lengths up to 36 can be measured as the distance between a mark and an end of the yardstick or as the distance between two marks. The two solutions are that you can place marks at the 1, 3, 6, 13, 20, 27, 31, and 35-inch positions, or at the 1, 5, 9, 16, 23, 30, 33, and 35-inch positions.

5 It takes a minimum of seven weighings to completely order eight weights when you know two partial orderings, $w_1 < w_2 < w_3 < w_4$ and $w_5 < w_6 < w_7 < w_8$. First, weigh the lightest from each of the two partially ordered groups (w_1 and w_5). The lightest of this weighing is the lightest weight. Next, if w_1 is lighter replace it with w_2; if w_5 is lighter replace it with w_6. The lighter weight of this second weighing is the second-lightest weight. Continue replacing the lightest weight of each weighing with the next-lightest weight from its partially ordered group. Following this procedure, you can always completely order the eight weights in seven weighings.

B onus. This problem asks for the equation of the flight path of a student pilot. The end points of the bridge at (-1,0), (1,0) and the pilot's initial position at (x_o, y_o) define a circle of equation $x^2 + (y - c)^2 = r^2$, where $c = (x_0^2 + y_0^2 - 1)/(2y_o)$ and $r^2 = c^2 + 1$. The visual angle is $\theta = \text{Arctan}\,[(c\text{-}y)/x]$. To maximize the instantaneous rate of increase of θ, the pilot must fly directly toward the center c of the circle. Then, we have $dx = -v\cos\theta dt$ and $dy = v\sin\theta dt$, where v is the speed of the plane, so that $dy/dx = -\tan\,\theta = (y\text{-}c)/x$. Substituting $c = (x^2 + y^2 - 1)/(2y)$ yields $dy/dx = (y^2 - x^2 + 1)/(2xy)$. Rearranging gives $2(y/x)dy - (y/x)^2 dx = d(y^2/x) = (1/x^2 - 1)dx$. Integrating yields $y^2/x = -1/x - x + 2A$; rearranging yields $(x - A)^2 + y^2 = A^2 - 1$. The path of the plane is then the arc of a circle with this equation. The initial condition yields $A = (x_0^2 + y_0^2 + 1)/(2x_o)$ and $r^2 = A^2 - 1$.

DOUBLE BONUS. You are asked to find and prove the extreme values L and H of $L \le \sin(3A) + \sin(3B) + \sin(3C) \le H$, where A, B, and C are the measures of the angles of a plane triangle. Substituting $(\pi - A - B)$ for C yields $L \le \sin(3A) + \sin(3B) + \sin(3A + 3B) \le H$. Taking the derivative and setting it equal to zero yields $3\cos(3A) + 3\cos(3A + 3B) = 0$. Thus, $A = (\pi - 3B)/6$. Since $\sin(3A) + \sin(3B) + \sin(3A + 3B)$ is symmetrical in A and B, two of the angles A, B, and C will be equal. If $A = B$, $(A, B, C) = (\pi/9, \pi/9, 7\pi/9)$, and $3\sqrt{3}/2$ is the maximum value. If $A = C$, $(A, B, C) = (\pi/3, \pi/3, \pi/3)$, and 0 is the minimum value. The case $B = C$ does not lead to a real triangle.

NEW FALL PROBLEMS

1 There are nine suspects in a crime. When questioned, each answered as follows:

JOHN: "Elvis is guilty."
GEORGE: "It was not Elvis."
RINGO: "I did it."
PAUL: "It was either Ringo or Tommy."
ELVIS: "George isn't telling the truth."
FABIAN: "Ringo is guilty."
CHUBBY: "It was not Ringo."
TOMMY: "It was neither Ringo nor I."
RICKY: "Tommy is telling the truth, and it wasn't Elvis either."

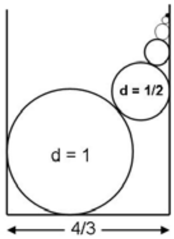
Only three of the nine suspects are telling the truth. That being so, who committed the crime?

—*Technology Review*

2 The long line of entrants for the marathon was told to number themselves from one upwards using the large supply of sticky-backed digits provided. All was going well, and I observed that my number was an even, three-digit, perfect square. The next person added one to my number, selected the appropriate digits, but then, unfortunately, stuck them on his back in the wrong order. The next person added one to that incorrect three-digit number, selected the appropriate digits, but then again stuck them on his back in the wrong order. The person after that added one to that incorrect three-digit number, selected the appropriate digits, but then stuck them on his back in the wrong order, and so it continued. Oddly, every third runner after me had a number which was a perfect square. I only realized that something was wrong when I overtook a runner with the same number as mine. What was my number?

—Susan Denham in *New Scientist*

3 An infinite number of cylinders are stacked in a trough as shown in the figure. If the diameters of the cylinders are 1, 1/2, 1/4, 1/8, etc., and the width of the trough is 4/3, then what is the height of the stack?



—*Technology Review*

4 Select three points at random from within a triangle. What is the expected distance of the nearest one from the base?

—**John W. Langhaar,** *PA A '33*

5 Your company makes long thin widgets of length L where L^2 is an integer. Each widget is shipped in a box having integral dimensions in length L, width W, and height H. The widget can be packed into the box in any one of three ways: 1) it can be packed parallel to any edge, but its length must equal the length of the box in that dimension, 2) it can be packed against any side of the box, but its length must equal the length of the diagonal along that side of the box, and 3) it can be packed into the box along the body diagonal, but its length must equal that of the box body diagonal. You discover that there are at least some widgets (for example, $L^2 = 23$) that cannot fit into any box using the above rules. Find all the values of L^2 representing widgets that cannot be packed into boxes using the above rules. Express your answer in terms of integers and variables that have only integral values.

—**John L. Bradshaw,** *PA A '82*

BONUS. On one of the holes of a golf course, the elevation of the tee is 40 feet higher than the elevation of the green. The horizontal distance between the tee and the cup is 250 yards. For an initial velocity of 232.5 ft/sec, at what angles could the ball be hit if the golfer wants the ball to land on the green 35 feet in front of the cup? Assume there is no wind and no spin on the ball. The ball has a diameter of 1.65 inches and a weight of 1.62 ounces. Assume the drag coefficient, C_D, is 0.25. Air resistance is given by $\rho_a C_D A v^2/2$, where $\rho_a = 0.075$ lb/ft^3 is the density of air, A is the cross sectional area of the ball, and v is its velocity. Use 32.088 ft/sec^2 for the acceleration due to gravity.

—*Adapted from Towing Icebergs, Falling Dominoes, and Other Adventures in Applied Mathematics* by Robert B. Banks

COMPUTER BONUS. A Smith number is a composite number, the sum of whose digits equals the sum of the digits of all its prime factors. The smallest Smith number is 4 = 2x2. The sum of the digits of 4 is 4, and the sum of digits of its prime factors is 2 + 2 = 4. Another example is 6,036 = 2x2x3x503; 6,036 has a digit sum of 15, and the sum of the digits of its prime factors is also 15. How many Smith numbers are there between 2 and 10,000?

—Patrick Costello & Kathy Lewis in *Mathematics Magazine* (June 2002)

Postal mail your answers to any or all of the Brain Ticklers to: **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT around mid-January. The method of solution is not necessary. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded. Jim will forward your entries to the judges, who are: **H. G. McIlvried** III, *PA Γ '53*; **D. A. Dechman**, *TX A '57*; **F.J. Tydeman**, *CA Δ '73;* and the columnist for this issue,

J. L. Bradshaw, *PA A '82*