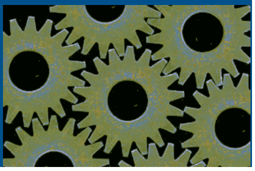


Brain Ticklers

RESULTS FROM SPRING 2006

Perfect

*	Baines, Elliot "Chip" A., Jr.	NY Δ '78
	Couillard, J. Gregory	IL A '89
	De Vincentis, Joseph W.	TX Γ '93
*	Voellinger, Edward J.	Non-member
*	Weinstein, Stephen A.	NY Γ '96

Other

*	Alexander, Jay A.	IL Γ '86
	Aron, Gert	IA B '58
*	Beaudet, Paul	Father of member
	Bernacki, Stephen E.	MA A '70
	Brana-Mulero, Francisco J.	PR A '74
	Brule, John D.	MI B '49
	Crawford, Martin	TN A '54
	Davisson, Roger C.	CA B '65
	Dower, Justin	Son of member
	Eddy, Sarah J.	DC B '01
*	Garnett, James M., III	MS A '65
	Golembiewski, Steven L., Jr.	PA B '90
	Hohmann, Lawrence A.	PA A '51
	Hollingsworth, David	CA P '83
	James, Catherine A.	Wife of member
	Jones, Donlan F.	CA Z '52
	Brown, Colin	Non-member
	Cannady, Nolan	Non-member
	Schneider, Megan	Non-member
	Karlsson, Rolf B.	MI Z '96
	Klaver, David S.	NY A '05
	Larson, M. Rhett	KS Γ '04
	Marks, Lawrence B.	NY I '81
	Marks, Ben	Son of member
	Marks, Noah	Son of member
	Marrone, James D.	IN A '87
*	Mayer, Michael A.	IL A '89
	Meerscheidt, Kyle	Husband of member
	Mir, Leon	NY A '59
*	Nabutovsky, Joseph	Father of member
	Nevins, Russell T.	MA B '77
*	Pecsvaradi, Thomas	PA Z '64
	Quintana, Juan S.	OH Θ '62
	Regula, D.W.	MI E '63
	Rentz, Peter E.	IN A '55
	Robillard, David J.	MD Γ '88
*	Schleehauf, Martin W.	NY N '79
	Schlotte, James F.	CA Ξ '82
	Schmidt, V. Hugo	WA B '51
	Speller, David	OH B '79
*	Spong, Robert N.	UT A '58
	Stribling, Jeffrey R.	CA A '92
*	Strong, Michael D.	PA A '84
*	Thaller, David B.	MA B '93
*	Upshur, John I.	VA A '83
	Valko, Andrew G.	PA Λ '80
	Vogt, Jack C.	OH E '56
	Yee, David G.	NJ B '04
	Young, Roger A.	OR A '75
	Zison, Stanley W.	CA Θ '87

* Denotes correct bonus solution

SPRING REVIEW

The Spring Tickler missed most often was No. 3 about the coin-tossing game, which proved to be more difficult than the Bonus about three random points within a cone. Several entries missed perfect status by missing half of a two-part problem.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter 2007 BENT. Meanwhile, here are the answers:

1 This problem asked for an expression for the first number of a sequence of at least N consecutive composite integers in terms of N. Start with $(N + 1)!$, where $n!$ is the product of all integers from 1 through n. Now, $(N + 1)! + i$ will be divisible by i for $i = 2$ to $i = N + 1$, a range of N terms. Thus, $(N + 1)! + 2$ is the first term of a sequence of at least N consecutive composite integers.

Credit was given for other correct answers.

2 Sam (S) is the archivist (a), Ronnie (R) the baker (b), Peter (P) the clergyman (c), and Quentin (Q) the doctor (d). First, determine the scores of the bridge rubbers from the facts that d ended with +\$5, P ended -\$19, and the largest rubber was 800. The only scores consistent with these facts are 400, 700, and 800. Next, look at the possible scores of d's matches. There are only six ways to arrange

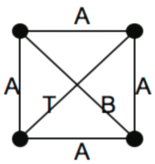
pairing	1	2	3	4	5	6
da/bc	8	4	8	4	-7	-7
db/ac	4	8	-7	-7	8	4
dc/ab	-7	-7	4	8	4	8
total score						
a	11	3	11	3	-19	-19
b	3	11	-19	-19	11	3
c	-19	-19	3	11	3	11
d	5	5	5	5	5	5

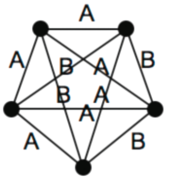
the scores of the rubbers so that d finishes +\$5 and the two unknown scores come out as +\$3 and +\$11 (see table above). One quickly eliminates 4 of the 6 columns; the two remaining columns (nos. 2 and 5) each have ac losing the largest rubber; therefore a and c are brothers. Then, from "Quentin's father's brother" and "clergyman's brother," one concludes that Q is not c or a; Q must be b or d. Finally, use the fact that the PR partnership fared better than PQ to determine that -400 can't be the score of PQ's match. Thus Q is not b; therefore Q is d. Since S did worse than b, S must be a or c, and finally R must be b. This means that c and a are P and S and that P and S are brothers. We also know that d is older than a and P is older than S. Thus a=S, b=R, c=P, and d=Q.

3 The numerical equivalent of WIT x WILL = THIRST is 235 x 2311 = 543085. A detailed explanation is omitted due to space limitations.

4 Set up the unit square so that it lies in the (x, y) plane with corner D at the origin. Then, the coordinates of vertices A, B, C, and D are (1, 0), (1, 1), (0, 1), and (0, 0), respectively. Let point P have coordinates (x, y). Then, we have $u^2 = (x\text{-}1)^2 + y^2$, $v^2 = (x\text{-}1)^2 + (y\text{-}1)^2$, $w^2 = x^2 + (y\text{-}1)^2$, and $z^2 = x^2 + y^2$, where u, v, w, and z are the distances from P to A, B, C, and D, respectively. Substituting these values for u^2, v^2, and w^2 into $u^2 + v^2 = w^2$ gives $(x\text{-}1)^2 + y^2 + (x\text{-}1)^2 + (y\text{-}1)^2 = x^2 + (y\text{-}1)^2$, which simplifies to $y^2 = -x^2 + 4x - 2$. Substituting this value for y^2 into the expression for z^2 and simplifying gives $z^2 = 4x - 2$. Thus z will be a maximum when x is a maximum, but since y^2 is non-negative, then $-x^2 + 4x - 2$ is also non-negative. Setting this expression equal to 0 and solving, we get $x = 2 + \sqrt{2}$ as the maximum value x can have. For this value of x, $y = 0$. Therefore, the maximum value of z, the maximum distance of P from D, is also $2 + \sqrt{2}$.

5 This problem asked for the maximum number of communities in a county that could be linked by bus, train, or airplane subject to various constraints. We found the answers to be either four or five communities, depending on conditions. Below are examples of networks with four and five communities.





Bonus. This problem asked for the minimum number of keystrokes to try every 4-digit combination of a cipher lock. Since each digit can be 0 through 9, the keypad then necessarily has to have ten keys numbered 0 through 9. One approach is to determine the minimal number for the simpler cases of 2-digit, then 3-digit combinations. For a 2-digit combination, one answer is 0-010203040506070809-1-1213141516171819-2-23242526272829-3-343536373839-4-4546474849-5-56575859-6-676869-7-7879-8-89-90, or 101 keystrokes. The dashes are inserted to suggest a pattern and so a generalization for 3-digit combinations. The minimal number of 3-digit combinations is 1002, a minimal sequence starts 0-001002003004005006007008009011012013014015016017...098099-1-112113114115...198199-2-.... For a 4-digit combination, the minimal number of keystrokes is clearly 10,003. Start with 3 keystrokes; then each new keystroke forms a new combination. Since there are 10,000 possible combinations, this will require 10,000 keystrokes, giving a total of 10,003. For an N-digit combination, the answer is $10^N + (N\text{-}1)$. These answers are specific cases of de Bruijn sequences. A de Bruijn sequence $B(k,n)$ is a cyclic sequence from a given alphabet A of size k for which every possible subsequence of length n in A is present exactly once.

Double Bonus. When $a(n+1)= a(n)^2+2b(n)c(n);\ \ 0\leq a(0),b(0),c(0)<1;$ $a(0)+b(0)+c(0)=1;$ and there are similar relations for $b(n)$ and $c(n)$; then the limit of $a(n)$ as n approaches infinity is *1/3*. A proof is omitted here due to space limitations but will be supplied upon request.

FALL PROBLEMS

1 *The wife of a man who grew barley*
Was also the sister of Charlie.
Her neighbor grew hay
And was married to Ray,
And one of these girls was named Carly.

The girl who was married to Wayne
Lived next to the farm that grew grain.
She liked to eat celery
That was grown by Valerie,
And she weighed 80 more pounds than Jane.

The woman whose husband grew dill
Was never married to Bill.
Before Jane married Benny
And Ray married Jenny,
She went out drinking with Jill.

Who is married to whom, and what does each couple grow? Only one couple has rhyming names.

—*Technology Review*

2 Uncle Wilbur was rich enough in life to command the boundless devotion of his family. However, some of them he despised, and some he merely disliked. Therefore, he repaid them by leaving his entire estate to be divided equally among the 581 inmates of a home for retired greyhound racing dogs. The sum involved ran to five figures and, when divided, gave each dog a whole number of dollars. By coincidence, the exact number of relatives he despised happens to equal the first two digits of the value of his estate, and the third digit is the number he merely disliked. The last two digits equal the number of his kinsfolk, that is the sum of those he despised and those he merely disliked. How much money did each dog get?

—*Adapted from Martin Hollis*

3 A 3 m by 3 m rug that is 1 cm thick is rolled up on a wooden cylinder 10 cm in diameter. Assume that the first wrap leaves a small gap between the rug and the cylinder (assume that the bottom surface of the rug forms a straight line between the wooden cylinder and the edge of the rug), but that there are no other gaps in the roll. What is the distance from the outer edge of the rolled rug to the center of the wooden cylinder?

—**Oscar C. Bascara,** *NY Δ '90*

4 A spherical plum pudding of radius R contains N infinitely small plums, randomly distributed. What is the expected distance of the nearest one from the surface?

—*W.A. Whitworth, 1901*

5 A flexible string of length L hangs from the bottom of a spherical fixture of radius R. The string swings back and forth in a plane. What is the maximum area that the string can sweep out? (Assume $L < \pi R$.)

—*adapted from Howard P. Dinesman*

Bonus. A needle of length l is thrown randomly onto a ruled grid. The horizontal spacing of the grid is d_1 and the vertical spacing of the grid is d_2. What is the probability that the needle will cross either a horizontal or a vertical line (or both)? Consider only the case where $l<d_1<d_2$.

—*adapted from P.S. Laplace, 1821*

Double Bonus. It has been said that if enough monkeys were given enough time with enough typewriters, they would eventually reproduce all of the works of Shakespeare. Suppose that there are 10,000,000 monkeys, each typing the 26 lower case letters plus the space bar at random, at 10 characters per second. What is the expected time interval between occurrences of the sequence "wherefore art thou romeo"?

—**Byron R. Adams,** *TX A '58*

Mail your answers to any or all of the Brain Ticklers to: **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT in early January. We also welcome any interesting new problems that may be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are: **H.G. McIlvried** III, *PA Γ '53*; **D.A. Dechman**, *TX A '57*; **F.J. Tydeman**, *CA Δ '73;* and the columnist for this issue,

J.L. Bradshaw, *PA A '82*