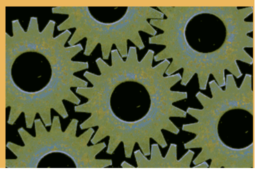

Brain Ticklers

RESULTS FROM SPRING 2005

Perfect

	Baines, Elliot "Chip" A., Jr.	NY	Δ	'78
	Couillard, J. Gregory	IL	A	'89
*	Garnett, James M.	MS	A	'65
*	Glysing-Jensen, Jesper	PA	A	'87
	Hess, Richard I.	CA	B	'62
	Karlsson, Rolf B.	MI	Z	'96
*	Kimsey, David B.	AL	A	'71
	Larson, M. Rhett	KS	Γ	'04
	Lonz, Gary J.	Non-member		
	McCormick, Michael J.	Son of member		
	Mellinger, Daniel W.	NC	A	'07
	Nabutovsky, Joseph	Father of member		
	O'Dorney, Evan	Non-member		
*	Pecsvaradi, Thomas	PA	Z	'64
	Rentz, Peter E.	IN	A	'55
*	Schleehauf, Martin W.	NY	N	'79
*	Schmidt, V. Hugo	WA	B	'51
	Spong, Robert N.	UT	A	'58
*	Vogt, Jack C.	OH	E	'56
	Yee, David G.	NJ	B	'04
	York, Jeff	NC	A	'85

Other

*	Alderson, William S.	MI	E	'43
	Alexander, Jay A.	IL	Γ	'86
	Aron, Gert	IA	B	'58
	Bachmann, David E.	MO	B	'72
	Bohdan, Timothy E.	IN	Γ	'85
	Brana-Mulero, Francisco J.	PR	A	'74
	Brule, John D.	MI	B	'49
*	Christenson, Ryan C.	UT	B	'93
*	Creutz, Michael J.	CA	B	'66
	Creutz, Edward C.	PA	Γ	'36
	Doniger, Kenneth J.	CA	A	'77
	Eiseman, Laurence H., Jr.	NY	Γ	'52
*	Fulara, Steven P.	IL	Z	'98
	Galer, Craig K.	MI	A	'77
	Gilliam, Grant	NC	Γ	'05
	Harvey, Arthur J.	OH	A	'83
	Jones, Donlan F.	CA	Z	'52
	Jordan, R. Jeffrey	OK	Γ	'00
	Lalinsky, Mark A.	MI	Γ	'77
	Moore, Campbell A.	MI	A	'64
*	Peithman, Harlan W., Jr.	IL	A	'53
	Quintana, Eric E.	TX	Δ	'84
	Rasbold, J. Charles	OH	A	'83
	Shaad, David L.	OH	Γ	'87
	Snyder, M. Duane	IA	B	'63
	Stadnik, Theodore	OH	Λ	'04
*	Stribling, Jeffrey R.	CA	A	'92
	Strong, Michael D.	PA	A	'84
	Summerfield, Steven L.	MO	Γ	'85
	Svetlik, Frank	MI	A	'67
*	Thaller, David B.	MA	B	'93
	Thrush, Thorle	OH	Γ	'48
	Tredeau, Frank	MA	E	'86
	Valko, Andrew G.	PA	Λ	'80
*	VanShaar, Steven R.	UT	Γ	'00
*	Voellinger, Edward J.	Non-member		
	Wolff, Nicholas L.	NE	A	'00

* denotes correct bonus solution

SPRING REVIEW

The most difficult of the regular Spring problems was No. 5, concerning the largest known prime in 2004; more than 33% of the answers were incorrect. The bonus problem about throwing an ellipse onto ruled parallel lines proved to be difficult; only ~38% of the received answers were correct.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter '06 BENT. Meanwhile, here are the answers:

1 In this problem, we asked for not only the results of the round robin chess tournament, but also the occupations of the players. The tournament results are as follows (2=Win, 1=Tie, 0=Loss):

Player	A	B	C	D	E	Score
A hairdresser	-	1	2	2	0	5
B financier	1	-	2	0	1	4
C interpreter	0	0	-	1	0	1
D journalist	0	2	1	-	0	3
E ghostwriter	2	1	2	2	-	7

2 You were asked to find the maximum size of a set of N distinct positive integers, the sum of any K of which is prime, for the cases when K = 2, 3, 4, and 5. The first case to try is N = K; in this case (1, 2, 3, …, K-1, X) can be shown to be a solution set, where 1 through K-1 are consecutive integers and X, greater than K-1, causes the sum of the set to be prime. When N is greater than K, it is useful to compute the sums *modulo* K. When any sum of K integers is zero *modulo* K, then they are a multiple of K and hence not prime. By looking at the sums *modulo* K, you can show that (K, N) = (2, 3), (3, 5), (4, 5), and (5, 9) are not possible. Then one can construct solution sets for (K, N) = (2, 2), (3, 4), and (4, 4). The K = 5 case is much harder. We used a computer to find a solution for (5, 7). The situation for (K, N) = (5, 8) is still open; we have neither a proof that it cannot be done nor a valid solution set. We intend to be lenient in grading. The following list presents the largest sets that we found for each case.

K	N	example
2	2	{1,2}
3	4	{1,3,7,9}
4	4	{1,2,3,5}
5	7	{1,3,13,43,253,751,979}

3 The smallest anti-magic square (all magic sums different) consisting of nine positive, not necessarily all different integers, that we found was:

1	1	1
1	2	4
3	1	5

As judged by the "smallness" criteria, in decreasing order of priority, the largest integer used is 5, the sum of the integers used is 19, and the sum of the integers in the first row is 3.

4 The problem asks for the rational values that may be assumed by the expression $R = \sqrt{(N/7)^2 + 5{,}036}$, for N a positive integer. R can assume only three values: 61,692/7; 1,260; and 1,308/7 when N is 61,690; 8,806; and 1,210, respectively. If you express R as P/Q, where P and Q are positive integers, then the expression for R can be restated as $7P = Q\sqrt{N^2 + 246{,}764}$. This means that if $N^2 + 246{,}764$ is a perfect square, then the equation in P and Q will be solved for Q = 7 and $P^2 = N^2 + 246{,}764$. This later expression can be rewritten as (P + N)(P - N) = 246,764. Note that since 246,764 is even, both P + N and P – N must also be even. So, find all of the divisors (D_1) of 246,764 that leave an even quotient (D_2). The only three divisor pairs {D_1, D_2} are {2; 123,382}, {14; 17,626}, and {98; 2,518}. Finally note that $N = (D_2 - D_1)/2$ and $P = (D_2 + D_1)/2$. This yields N = 61,690; 8,806; and 1,210, and R as stated above.

5 This problem asked for which of two different income averaging schemes, bottom-up or top-down, would the richest and the poorest income brackets in Puevigi prefer. Number the income groups from one to ten. When neighboring income groups are averaged in order from poorest to richest, the resulting nine income groups have values of {1.5, 2.25, 3.125, …, 9.00195}. When the averaging proceeds from richest to poorest, the resulting nine income groups have wealth values of {9.5, 8.75, 7.875, …, 1.99805}. Thus both the richest and the poorest income groups would prefer the second scheme that works from the top down.

BONUS. You were asked to get 8 liters in a 10-liter container, 5 liters in a 6-liter container, and 2 liters in a pail, starting with a full 15-liter container. The best solution we know involves 15 transfers, as shown in the table. Since the pail has unknown capacity, the best strategy is to get 2 liters into the pail first and then to leave it alone. This takes a minimum of 5 transfers. Then it takes 10 more to get 5 liters in the 6-liter container and 8 liters in the 10-liter container. The table below shows the contents of the containers after each transfer.

15L	10L	6L	Pail	
15	0	0	0	=> START
9	0	6	0	
9	6	0	0	
3	6	6	0	
3	10	2	0	
13	0	2	0	also: (3, 10, 0, 2)
13	0	0	2	also: (3, 4, 6, 2)
7	0	6	2	
7	6	0	2	
1	6	6	2	
1	10	2	2	
11	0	2	2	
11	2	0	2	
5	2	6	2	
5	8	0	2	
0	8	5	2	=> END

DOUBLE BONUS. In this problem, you were asked to prove that the three points of intersection of the three pairs of lines that are the external tangents of three circles, each circle externally tangent to two others, lie on a straight line. Let A be the center of the largest circle (of radius R_a), B the center of the middle circle (of radius R_b), and C the center of the smallest circle (of radius R_c). Let P be the intersection point of the tangents to circles A and B, Q be the intersection point of the tangents to circles A and C, and R be the intersection point of the tangents to circles B and C. Construct a fourth circle with center D such that its radius is the same as the radius of the smallest circle (R_c) and it is tangent to both the external tangents of circles A and B. Then:

$$PD/PA = R_c/R_a = QC/QA$$

Hence, DC is parallel to PQ. In the same way, DC is parallel to PR. Thus, segment PQ is part of line PR.

The following *intuitive* solution is attributed to John Edson Sweet (*You are a Mathematician* by D. Wells). Imagine having three different spheres touching one another and sitting on a plane, say plane #1. Then there exists a second plane, plane #2, tangent to the three spheres but covering them from above. Planes #1 and #2 are bound to have a common line l, the line of their intersection. Finally, note that the bisector plane of the solid angle formed by planes #1 and #2 passes through the centers of all three spheres. This is the plane, plane #3, that contains the three externally tangent circles and the six external tangents. By definition, plane #3 passes through line l, and line 1 contains the three points of intersection.

NEW FALL PROBLEMS

1 Solve the following cryptic addition:

A/DE + B/FG + C/HJ = 1

where each letter represents a different digit and none of them is zero. DE, FG, and HJ are each two-digit integers, e.g. DE = 10 * D + E. We want the solution in which A < B < C.

—*Technology Review*

2 Rex lies a lot. In fact, he tells the truth on only one day of the week. One day, he said, "I lie on Mondays and Tuesdays." The next day, he said, "Today is either Thursday, Saturday, or Sunday." The next day, he said, "I lie on Wednesdays and Fridays." On which day of the week does Rex tell the truth?

—*Mind Puzzlers* by George J. Summers

3 What is the dihedral angle between two faces of a regular tetrahedron? Express your answer exactly using integers and trigonometric and algebraic functions. —Paul J. Blatz

4 Here is the situation on a game show. As three contestants enter the stage, a red or blue hat is placed on each one's head, depending on the flip of a fair coin. Each contestant can see the other's hats but not his own, and they cannot communicate with each other in any way. They are told that if one or more of them can identify the color of hat he or she is wearing and no one gives a wrong answer, each will win a prize, but if no one answers or if one or more give a wrong answer, they will get nothing. However, before the hats are put on them, they are allowed to consult with each other. If they adopt the optimal strategy, what is their probability of winning?

—**Dr. Todd D. Ebert**, *PA B '91*
University of California-Irvine

5 Assume that you have a frictionless hemisphere of radius R with its flat side fastened to a perfectly level table. You place a very small object of mass *m* infinitesimally off center on top of the hemisphere. At what height above the table does the object leave the hemisphere, and how far from the center of the hemisphere does it land?

—*Technology Review*

BONUS. Find the equation traced by the locus of the focus of the parabola $y = ax^2$ as this curve is rolled without slipping against the x-axis.

— **John L. Bradshaw**, *PA A '82*

DOUBLE BONUS. Consider an infinite square lattice of points with integer coordinates on a plane. If two lattice points are chosen at random, what is the probability that the straight-line segment joining the two lattice points will not pass through any other lattice points?

—*The Joy of Pi* by David Blatner

Mail your answers to any or all of the Brain Ticklers to: **Jim Froula, Tau Beta Pi, P. O. Box 2697, Knoxville, TN 37901-2697** or email plain text to: *BrainTicklers@tbp.org*. The cutoff date for entries to the Fall column is the appearance of the Winter BENT around early January. The method of solution is not necessary. We welcome any interesting new problems that might be suitable for use in the column. The Double Bonus is not graded. Jim will forward your entries to the judges, who are:
H. G. McIlvried III, *PA Γ '53;*
D. A. Dechman, *TX A '57;*
F. J. Tydeman, *CA Δ '73*;
and the columnist for this issue,
J. L. Bradshaw, *PA A '82.*