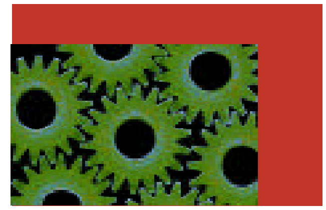

Brain Ticklers Celebrates 50th Year!

FIFTIETH ANNIVERSARY

The first Brain Tickler in THE BENT appeared in 1951 (a couple of months before this columnist was born). We continue our observance of the 50th anniversary by repeating below, as the snowfall problem, one of the problems that was used in 1951.

Here are a few statistics from 1971-80, the third decade of our existence: 1,838 entries from 960 people were submitted during this period, compared to an average of 1,467 entries per decade for the previous two decades.

The highest number of perfect solutions came from:

Byron R. Adams, TX Α '58;
James L. Griggs Jr., OH Α '56;
R. Wilson Rowland, MD Β '51.

Submitters with the most correct Bonus solutions were (including a tie for third):

Byron R. Adams, TX Α '58;
R. Wilson Rowland, MD Β '51;
Robert H.A. Meyer, NJ Α '53;
William A. Conwell, PA Γ '29.

Two of the above became Brain Tickler judges. Mr. Meyer is a repeat from 1961-70 decade statistics.

SPRING REVIEW

The Spring set turned out to be relatively easy. The hardest problem was No. 5 about arranging pentominos on a chessboard, with fewer than half of the responders getting the correct answer. In fact, No. 5 proved to be as difficult as the Bonus problem about the volume of sky reachable by an antiaircraft gun.

SUMMER ANSWERS

Those who responded to the Summer Ticklers will be acknowledged in the Winter column. In the meantime, here are the answers.

1. The hour and minute hands of a clock form a straight line at a little past 8 a.m. and a little before 3 p.m. You were to find the elapsed time between these events. Clock hands are aligned at 6 a.m., 6 p.m., and ten times in between, for a between-alignments time period of 12/11 hour. Some contemplation shows that there must be six of these periods between our events. So the elapsed time is 72/11 hours or 6 hours, 32 minutes, 43 7/11 seconds.

2. The number of palindromic integers in the range from 1 to 10^{10} is 199,998. There are 9 from 1 to 9, 9 from 10 to 99, 90 from 100 to 999, 90 from 1,000 to 9,999, etc., for a total of 2(99,999) = 199,998.

3. GO x FLY = KITES
27 x 594 = 16038
where FLY is divisible by GO.

4. On the first day of spring, the tall Indians in Ecuador can see the sun 3.6 seconds longer than their short neighbors can. Let θ be the angle subtended at the center of the Earth by the distance from Indian to horizon. Then $\cos\theta = R/(R + h)$ where R = 6,378 km is the radius of the Earth and h (either 1.4 or 2 m) is the height of the Indian's eyes. For small angles $\cos\theta = 1 - (\theta^2)/2$, giving $\theta = \mathrm{sqrt}(2h/R)$. The resulting angles, plus the Earth's rate of rotation of 360°/day, lead to a time difference of 3.556 seconds.

5. The 2 x 3 arrays of stamps shown below permit all integer postal rates from 1 to 36 to be met using either a single stamp or a set of connected stamps. The theoretical maximum is 40—the total number of different stamp combinations (6 singles, 7 pairs, etc.)—but there is no arrangement that achieves greater than 36.

1	2	15
4	6	8

1	3	17
8	2	5

Bonus. The 10 beads (two each of five different colors) can be arranged to form 5,736 different bracelets. Assume that every bracelet contains two red beads. For n pairs of beads, the number of linear permutations of the non-red beads is $P(n\text{-}1) = (2n\text{-}2)!/2^{(n\text{-}1)}$.

To form a bracelet, append one red bead at the start of a permutation, insert the second red bead at any position up to the middle, and join the last bead to the first bead. That is, the red beads can be inserted so that there are i other beads between them, where $i = 0, 1, 2,..., n\text{-}1$.

Consider the case where $i = 0$; that is the two red beads are appended to the start of each of the $P(n\text{-}1)$ permutations. However, this does not give $P(n\text{-}1)$ bracelets, as not all these bracelets are unique. There are two types of permutations, those that are palindromes (e.g., YOBGGBOY) and those that are not. Each palindromic permutation, of which there are $(n\text{-}1)!$, will give a unique bracelet; whereas, a nonpalindromic permutation and its reverse will give the same bracelet. Thus, the number of unique bracelets with $i = 0$ is $R(n) = [P(n\text{-}1) - (n\text{-}1)!]/2 + (n\text{-}1)! = [P(n\text{-}1) + (n\text{-}1)!]/2$. The same result holds for $i = 1$ through $n\text{-}2$.

For $i = n\text{-}1$, that is with the red beads opposite each other, three types of symmetry need to be considered, e.g., YOBGGBOY, YOBGYOBG, andYOOYBGGB; the number of permutations of each type is $(n\text{-}1)!$. For each symmetric permutation, there is another permutation, namely GBOYYOBG, GBOYGBOY, and BGGBYOOY, respectively, for the above examples, that gives the same bracelet when the red beads are added. For the nonsymmetric permutations, in all cases there will be four different permutations that give the same bracelet. Thus, the total number of bracelets with the red beads opposite is $Q(n) = [P(n\text{-}1) - 3(n\text{-}1)!]/4 + 3(n\text{-}1)!/2 = [P(n\text{-}1) + 3(n\text{-}1)!]/4$.

Therefore, the total number of unique bracelets is $T(n) = Q(n) + (n\text{-}1)R(n)$, which leads to $T(n) = (2n\text{-} 1)!/2^{(n+1)} + (2n+1)(n\text{-}1)!/4$. This formula gives $T(1) = 1$; $T(2) = 2$; $T(3) = 11$; $T(4) = 171$; $T(5) = 5{,}736$; $T(6) = 312{,}240$; etc. It is easier to see the patterns that develop by starting with a small number of pairs of beads and working your way up.

Double Bonus. One of the chemist's 12 weights was either too light or too heavy, giving 24 possibilities. Each of his three weighings on the balance can be left-heavy, balanced, or right-heavy; this gives $3^3 = 27$ possible outcomes, which is enough. Shown below is one way to conduct the weighings. This would seem to give 3 equations with 12 unknowns, but it works. For example, weighings of right-heavy, balanced, and right-heavy determine that weight 3 is light. Note that all three weighings being balanced would indicate that a 13th weight is the odd one, although its condition of being heavy or light could not be determined.

1 2 3 4 vs. 5 6 7 8
1 5 6 9 vs. 2 7 A B
3 7 8 A vs. 2 6 B C

NEW PROBLEMS

1. During the night, snow began to fall at a constant rate. Sometime later, the highway crew began removing snow from the roads with their one plow. The rate of snow removal (by volume) was constant, and the plow moved two miles in the first hour and one mile in the second hour. How long had it been snowing before the crew started?

—**John R. Osborn,** *IN* A *'50*

2. George, John, Arthur, and David are married, not necessarily respectively, to Christine, Eve, Prudence, and Rose. They remember that at a party years ago various predictions had been made. George had said that John would not marry Christine. John had said that Arthur would marry Prudence. Arthur had predicted that whomever David married, she would not be Eve. David, who at that time was more interested in sports than matrimony, had predicted that the Cowboys would win the next Super Bowl. The only one to predict correctly was the man who married Prudence. Who married whom?

—Adapted from *Brain Puzzler's Delight* by E.R. Emmet

3. With different letters being different digits, the same letter being the same digit throughout, and base 10, solve:
ADAM+AND+EVE+ON+A=RAFT
with ADAM and EVE being as close as possible.

—*Mathematical Puzzles for Beginners & Enthusiasts* by Geoffrey Mott-Smith

4. Uncle George (who was younger than 100 years old when he received this) found this old message that had been buried in his inbox:

Sept. 1973

Dear George,
When I was last home you were wondering whatever became of Pongo Pongleton, who used to go swimming with you in 1947. Well, I bumped into him in Sweatipore last week. He told me he moved up here a couple of years ago with his wife and two children to study the sex life of the wombat or some such prurient nonsense. By an odd chance the present ages of his children (in whole years) multiply to your present age. The girl is a bookworm, but the boy is mad about guns. He has just broken the largest window in the house with his dad's air-rifle. I recall you doing just the same when you were exactly his age. It's a small world.

Yours ever,
Henry

George could remember nothing at all about the window incident referred to in the letter. But, using only Henry's letter and his own date of birth, he deduced the exact year in which it must have occurred. What was George's precise date of birth (year, month, and day)?

—Martin Hollis

5. Drawing the five diagonals in a convex pentagon results in a five-pointed star. At each point of the star, between the sides of the star and the sides of the pentagon, are two angles—i.e., angles exterior to the star and bounded by the pentagon. What is the sum, in degrees, of the measures of these 10 angles?

—*Technology Review*

Bonus. A student starts with the familiar series 1 + 1/2 + 1/4 + 1/8 + ... He then takes the average of each adjacent pair of terms and inserts it between the terms to obtain the new series 1 + 3/4 + 1/2 + 3/8 + 1/4 + ... He divides this by two, because there are now twice as many terms as before. He repeats the process indefinitely. What exact limit will the series approach?

—Litton Industries "Problematical Recreations"

Double Bonus. Decipher the following phrases (where each capital letter is the first letter of a word, e.g., 10 = Decimal Digits):

1 = W. on a U.
2 = S. to an I.
3 = B. M. (S. H. T. R.)
4 = Q. in a G.
5 = D. in a Z. C.
6 = S. on a C.
7 = W. of the A. W.
8 = S. on a S. S.
9 = P. in the S. S.
10 = D. D.
11 = P. on a F. T.
12 = S. of the Z.
13 = S. on the A. F.
18 = H. on a G. C.
24 = H. in a D.
26 = L. of the A.
29 = D. in F. in a L. Y.
32 = D. F. at which W. F.
40 = D. and N. of the G. F.
54 = C. in a D. (with the J.)
57 = H. V.
64 = S. on a C.
88 = K. on a P.
90 = D. in a R. A.
200 = D. for P. G. in M.
1000 = W. that a P. is W.
1001 = A. N.

—Adapted from *The Crucible*

The judges are:
H.G. McIlvried III, PA Γ '53;
R.W. Rowland, MD Β '51;
D.A. Dechman, TX Α '57;
and the columnist for this issue:
Fred J. Tydeman, *CA Δ '73.*