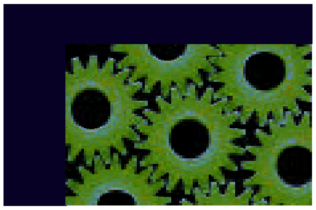


BRAIN TICKLERS

## SPRING REVIEW

The most difficult problem to judge in the Spring set was No. 1 about the depth of the water in the hemisphere.

## SUMMER ANSWERS

Readers' entries for the Summer problems will be acknowledged in the Winter BENT. Meanwhile, here are the answers:

**1** Represent Meek's telephone number by MEEK, Humble's by EXYM, and Lowly's by MXYE. They have the relationship of EXYM - MXYE = MEEK. Inspection shows that E = 9 and M= 4, requiring that K = 5. Thus Meek's number is 4995.

**2** For a probability 0.300 hitter, a probability tree shows that the probabilities of getting exactly 2 and 3 hits out of 10 are 0.2335 and 0.2668.

Another way to get 0.2335 is from the formula for getting $h$ hits in $a$ at bats with $b$ batting average is $P=C(a,h)b^h(1-b)^{(a-h)} = (10!/8!2!)(.3^2)(.7^8)$. For 0.200, they are 0.3020 and 0.2013. It is given that Mike got 3 hits out of 10 and John got 2.

Thus, the probability that Mike at 0.300 got 3 and John at 0.200 got 2 is $P32$ = 0.2668x0.3020=0.08058.

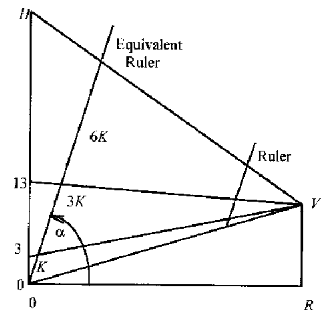
The probability that Mike at 0.200 got 3 and John at 0.300 got 2 is $P23$ = 0.2013x0.2335 = 0.04700. Since these are the only two possible outcomes, the probability of Mike being the 0.300 hitter is $P32/(P32+P23)$ = 12/19 = 0.6316.

**3**

| | |
|---|---|
| OLD | 394 |
| SALT | 8091 |
| TOLD | 1394 |
| TALL | 1099 |
| TALES | 10978 |

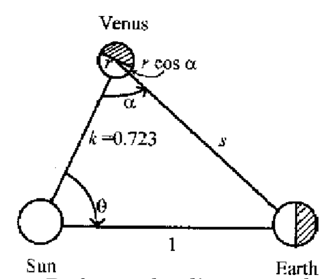
**4** The 100 coins of amount $2.73 must comprise 4 quarters, 5 dimes, 8 nickels, and 83 pennies.

**5** The height of the bridge tower is 39 meters. Per the diagram, let $H$ be the height of the tower, $V$ be the height of the viewer, $R$ be the distance from bridge to viewer, α be the angle of the ruler, and $K$ be the proportionality factor for the ruler. Equations for the sightings of the heights of 3, 13, and $H$ then are:




$(K\sin\alpha-(VK\cos\alpha)/R)/3 = (R-K\cos\alpha)/R$
$(4K\sin\alpha-(4VK\cos\alpha)/R)/13=(R-4K\cos\alpha)/R$
$(10K\sin\alpha-(10VK\cos\alpha)/R)/H=(R-10K\cos\alpha)/R$

Five unknowns cannot be determined from the three equations. When the equations are combined, however, two of the unknowns cancel out, leaving the solution of $H$ = 39m.




**Bonus.** Refer to the diagram, where 1, $k$, and $s$ are in astronomical units. The problem is to determine the angle θ for which Venus appears to be brightest. Half of Venus is lit by the Sun, of course. From Earth, the light from Venus appears as a semicircle of area $\pi r^2/2$ plus or minus a semi-ellipse of area $\pi r(r\cos\alpha)/2$. The normalized light thus is $L = (1+\cos\alpha)/2$. The perceived brightness follows the inverse square law such that $B = L/s^2$. From the cosine law, $\cos\alpha=(s^2+k^2-1)/2sk$.

Substitute this into the equation for $B$, and rearrange to get

$B$=1/(4$ks$)+1/(2$s^2$)-(1-$k^2$)/(4$ks^3$) = $Ax$+$Dx^2$-$Cx^3$ where $A$=1/(4$k$), $D$=1/2, $C$=(1-$k^2$)/(4$k$), and $x$=1/$s$.

To find the maximum value of $B$, we differentiate with respect to $x$ and set the result to 0. This gives us a quadratic in $x$ that is easily solved using the quadratic formula, since the value of $k$ is known. Solving gives $x$ = 2.3212 and $s$ = 1/$x$ = 0.4308. Now, from the diagram, cosθ = $k$-$s$ cosα, so θ = 22.38°.

**Computer Bonus.** The largest hexadecimal number whose square contains each hexadecimal digit once and only once is FF6A AE41. The smallest such number is 404A 9D9B. The corresponding squares are FED5 B39A 4270 6C81 and 1025 648C FEA3 7BD9.

## NEW FALL PROBLEMS

**1** Smith, Jones, and Robinson make four statements each as follows:

Smith:
1. Jones owes me $10.
2. Robinson owes me $5.
3. All Robinson's statements are true.
4. All Jones's statements are untrue.

Jones:
1. I owe no money to Smith.
2. Robinson owes me $7.
3. I am British.
4. All Smith's statements are untrue.

Robinson:
1. I owe no money to anybody.
2. Jones is a Dutchman.
3. I always tell the truth.
4. Two of Jones's statements are true, and two are false.

One person made 4 true statements. Who? Find, for all of them, which statements are true and which are false.

— *Brain Puzzler's Delight* by E. R. Emmet

**2** Scalenia is a country bounded by three straight frontiers called $A$, $B$, and $C$, of course, each of a different length but each an exact number of kilometers long. The curious thing is that the length of $A$+$B$+$C$, and the lengths of $A$+$B$-$C$, $A$+$C$-$B$, and $B$+$C$-$A$, are all precisely square numbers of kilometers. If Scalenia has the smallest perimeter consistent with this curious fact, what is the length of each of its frontiers?

— *An Enigma* by Stephen Ainley

**3** A 12 hour AM/PM clock, whose display always shows six digits in the format hh:mm:ss, sits in a dark room. The digits of the LED display are formed by the familiar seven-element figures that look like dominos when all seven elements are lit. Between which two consecutive times is the change in the relative brightness of the display the greatest? Assume that, when it is lit, each element of each digit emits exactly the same amount of light. Also, assume that 0 is 6 segments lit, 1 is 2 seg, 2 is 5 seg, 3 is 5 seg, 4 is 4 seg, 5 is 5 seg, 6 is 6 seg, 7 is 4 seg, 8 is 7 seg, and 9 is 6 segments lit.

— *The Platinic Corner*

**4** Solve this cryptic division, where the right side is a repeating decimal. As usual, the same letter is the same digit throughout, different letters are different digits, and base - 10 is used: EVER/NNNNN= .ONAND ONAND. . . .

— Thomas McNelly via *Technology Review*

**5** An urn contains 10 balls labeled 0 through 9. You reach into the urn and pick three balls (without replacement) at random. What is the probability that you can form a three-digit prime number (leading zero not allowed) from the three balls?

— H.G. McIlvried, *PA* Γ *'53*

**Bonus.** In a uniform (direction and magnitude) gravity field, the fastest way for a frictionless particle to slide from A to B in a vacuum is along a cycloid. If A and B are 500 km apart and at the same elevation, the maximum depth of the cycloid is just over 159 km. If, however, there is a depth limitation of 50 km, describe the fastest curve. Also, with g = 9.8 m/s/s, how long (in seconds) does it take for a particle started at rest to go from A to B?

— David Gluss via *Technology Review*

**Double Bonus.** Devise a graphical method of locating point P in the plane of a given triangle ABC such that triangles PAB, PBC, and PCA have equal perimeters.

— John Rule via *Technology Review*

The judges are:
H.G. McIlvried III, *PA* Γ *'53;*
R.W. Rowland, *MD* Β *'51;*
D.A. Dechman, *TX* Α *'57;* and the columnist for this issue,
**F. J. Tydeman,** *CA* Δ *'73.*