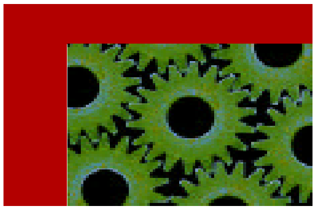

NEW TICKLERS

Welcome to the year 2000! The judges would like to thank Jim Froula and Judy Stewart for all their hard work and support for this column over the years and Martha James for the new format for our column.

The judges are: H.G. McIlvried, PA Γ '53; R.W. Rowland, MD B *'51; F. J. Tydeman, CA* Δ *'73; and the columnist for this issue,*
Don A. Dechman, *TX* A *'58*

Winter Introduction

We will not pose a brain tickler on whether the millennium starts with the year 2000 or 2001. Suffice it to say that the different appearance of these pages either is a celebration of the year 2000 or is a dress rehearsal for the real millennium next year.

1 A VCR feeds tape of uniform thickness onto a take-up reel at a constant velocity. A counter indicates a value proportional to the number of revolutions of the take-up reel. At time zero, the counter is set to zero. When the counter reads 1,000, the counter is increasing at a rate of 1 count per second. At 2,000 seconds, the counter is increasing at 2/3 count per second. Express time t as a function of the number of counts c.
— **Greg A. Qualls,** *KS* B *'80*

2 George's new office has a security lock in which you have to key in a series of digits, all different, before you can open the door. Unfortunately, he is having trouble committing this security number to memory. So far, he has memorized a number consisting of a selection of different digits from the security number. He made a note of this memorized number on the memo sheet on which he was given the security number, but he absent-mindedly left the paper lying around the house. When his wife found the paper containing the two numbers, she multiplied them together and found the product was a seven-digit number in which all the digits were the same. What is George's security number?
— Adapted from *New Scientist*

3 Thirty-two friends meet monthly for an eight-foursome golf scramble, after which they gather at the nineteenth hole to celebrate the birthday of those born that month. It so happens that one month there are no birthdays. Thinking that is highly unlikely, and needing an excuse for a party, they celebrate the event. For a random group of 32 people, what is the probability that there is at least one month in which no one has a birthday? Assume that the probability of a birthday being in any month is the same for all months.
— **John W. Langhaar,** *PA* A *'33*

4 A ladder of length L is standing vertically against a wall. As the bottom of the ladder is pulled away from the wall along the floor, the top of the ladder maintains contact with the wall. Find the equation of the boundary of the region swept out by the ladder as it is lowered to its horizontal position on the floor.
— *Technology Review*

5 A candle in the shape of a truncated right circular cone 45 cm high burns to extinction in nine hours. The bottom 3 cm take 20 minutes longer to burn than the top 3 cm. How many minutes will it take the top 3 cm to burn? Assume that the volume of the candle decreases at a constant rate as it burns.
— **Marcello J. Carrabes,** *MA* E *'50*

Bonus There are two integers, A and B, which are integers greater than 1 and less than 101. Neither Sam nor Pete knows what they are, but Sam knows their sum, and Pete knows their product. The following conversation takes place.

Pete: "I don't know what the numbers are."
Sam: "I knew that you did not know what the numbers are."
Pete: "Now I know what the numbers are."
Sam: "Then, so do I."

What are the values of A and B?
— *Don Holden*

Double Bonus A good approximation of pi using the digits from 1 through 9 each once, is:

$$\pi \approx 3 + (16 - 8^{-5})/(97 + 2^4)$$

Find a better approximation, again using the digits 1 through 9 each only once. The only operators allowed are addition, subtraction, multiplication, division, exponentiation, decimal points, and parentheses.
— **Richard I. Hess,** *CA* B *'62*

Summer Review

Submissions by e-mail got off to a good start with about 40 percent following that route.

Fall Answers

1 Aphrodite, Metis, and Hera scored 4, 1, and 5 points, respectively, in the Interesting Hobbies category. There is no solution if there are only four categories. There is a solution for five categories. Since Metis scored only 9 points, try 5 points for her first place and 4 third-place finishes at 1 point each. The total points must be at least 21 + 20 + 9 = 50, so try four points for second-place finishes. Hera must have three firsts, one second, and one third to amass 20 points. Aphrodite had one first and four seconds to total 21 points and win. Aphrodite's first place was in Beauty, so she finished second in Interesting Hobbies. Metis' first-place finish was in Wisdom, and, thus, finished third in Interesting Hobbies. Therefore, Hera must have finished first in Interesting Hobbies. The most possible categories is seven, with a 3-2-1 scoring system, due to the nine points for Metis requirement. But neither six nor seven categories yields a solution.

2 A beat B by a score of 3 to 1, C beat A 2 to 0, B and D tied 3 to 3, and C beat D 4 to 1. Each team plays the other teams once, so D played 1, 2, or 3 games. D must have played two games, since the sum of the games played by A, B, C, and D must be an even number. From D's scores, D must have lost one and tied one. A beat B by 3 to 1, and, thus, A lost its other game 0 to 2. B lost one and tied one, so C must have won its two games. So, C beat A 2 to 0. Then, B and D tied, and C beat D. But, B scored four goals total, and, thus, B and D tied 3 to 3. And, based on D's scores, C beat D 4 to 1.

3 The average life span is 72.70 years. People die randomly, such that all people aged 60 who die, die at age 60.5 on the average. And their probability of dying at age 60.5 is $(60/100)^8$ = 0.016796 of those who reach age 60. Set up a spread sheet with columns entitled age, average age, alive at start, die during year, and alive at end of year, and then have line entries for ages 0 to 100. Start with 1,000 people, and complete the table down through age 100. Then, get the weighted average of those who die during year times the average age to get 72.70 years as the average life span.

4 There are 13 ways to drill a straight hole from cube A to its diagonally opposite cube B. Label the cubes on the front face 1, 3, 4, and 2 in a clockwise manner and 5, 7, 8, and 6 on the back face, with 5 directly behind 1. The thirteen paths are:

1-8, 1-2-8, 1-3-8, 1-4-8, 1-5-8, 1-6-8, 1-7-8, 1-2-4-8, 1-2-6-8, 1-3-4-8, 1-3-7-8, 1-5-6-8, and 1-5-7-8.

5 The second player can always win by selecting her card to equal 15 minus the value of her opponent's previously chosen card. This causes the face-down sums to total 15, 30, 45, and then 60 at the end of the second player's turns.

Bonus The length is 63.03 meters. Represent the surface of the conical tree in one plane as a segment of a circle with a $\sqrt{10}$ meter radius and a circumferential arc length of 2π meters and an interior angle of $2\pi/\sqrt{10}$ radians. Twenty wraps of the tree are needed at 15 cm per wrap to total 3 meters. So, the total degrees for the 20 wraps is $4\pi\sqrt{10}$ radians. The shape of the light string up the flattened cone, using polar coordinates, is $dr/d\theta = \sqrt{10}/4\pi\sqrt{10} = 1/4\pi$ and $r = \theta/4\pi$. Using the polar coordinates formula for arc length and the standard polar coordinates notation for r and θ:

$$L = \int_0^{4\pi\sqrt{10}} [(\theta/4\pi)^2 + (1/4\pi)^2]^{0.5}\, d\theta\ .$$

Integration is straightforward and yields L = 63.03m.

Computer Bonus The smallest integer of the form $m2^n$ that fails the round-trip conversion is $1{,}017\text{x}2^{774}$. A relatively simple computer program can be written to check all integral values of m from 512 to 1,023, and for each such m, all integral values of n from 1 to 6,399 to find the stated answer. This solution can be verified with a hand-held scientific calculator. Its log is 236.0045376 which converts to $1{,}010.503\text{x}10^{233}$, which rounds to $1{,}011\text{x}10^{233}$, whose log is 236.0047512 which converts to $1{,}017.50022\text{x}2^{774}$, which rounds to $1{,}018\text{x}2^{774}$. There are five other such integers that are less than $512\text{x}2^{6399}$.