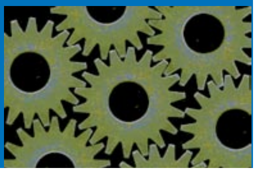

Brain Ticklers

RESULTS FROM

WINTER 2008

Perfect

Bachmann, David E.	MO B	'72
Barry, Michael S.	PA B	'08
Hess, Richard I.	CA B	'62
Marks, Lawrence B.	NY I	'81
Marks, Noah	Son of member	
Schmidt, V. Hugo	WA B	'51
Silver, Robert E.	NY P	'80
*Skowronski, Victor	NJ A	'71
*Smith, Charles J.	CT Γ	'09
*Spong, Robert N.	UT A	'58
Stribling, Jeffrey R.	CA A	'92
Summerfield, Steven L.	MO Γ	'85
Verkuilen, William W.	WI B	'92
Vogel, Jeffrey	Non-member	

Other

Aron, Gert	IA B	'58
Bernacki, Stephen E.	MA A	'70
Brule, John D.	MI B	'49
Chow, Jordon K.	NJ Γ	'05
Couillard, J. Gregory	IL A	'89
Hadizadeh, Rameen	KY A	'07
Harris, Kent	Non-member	
Jones, Donlan F.	CA Z	'52
Koehn, Thaddeus E.	RI B	'07
Larson, M. Rhett	KS Γ	'04
*Liu, Victor	CA B	'07
Midgley, James E.	MI Γ	'56
Quintana, Juan S.	OH Θ	'62
Rasbold, J. Charles "Chuck"	OH A	'83
Rentz, Peter E.	IN A	'55
Robertson, John A.	IL A	'65
*Routh, Andre G.	FL B	'89
Scholz, Gregory R.	PA B	'00
*Strong, Michael D.	PA A	'84
Svetlik, J. Frank	MI A	'67
Szostek, Renee	MI A	'87
Takahashi, Tsuyoshi	Non-member	
Thaller, David B.	MA B	'93
Van Houten, Karen J.	ID A	'76
*Venema, Todd M.	OH H	'08
Voellinger, Edward J.	Non-member	
Volder, Jack E.	TX B	'49
York, Jeffrey A.	NC A	'85

* Denotes correct bonus solution

WINTER REVIEW

Problem 5 about aligning dominos was the hardest regular problem, being missed by over half of the entries. The double bonus problem with Mozart being the mystery composer had more interest and more entries than usual.

SPRING SOLUTIONS

Readers' entries for the Spring problems will be acknowledged in the Fall BENT. Meanwhile, here are the answers:

1 The probability that an individual has his eyes closed is 6(0.1)/60 = 0.01. Therefore, the probability his eyes are open is 0.99, and the probability that 100 people all have their eyes open is $P_O = 0.99^{100} = 0.36603$. Now, the probability, P_C, that at least one person has eyes closed is $P_C = 1 – P_O = 0.63397$. For N pictures, the probability that all have at least one person with closed eyes is P_C^N. We want $0.63397^N = 0.05$. N = ln0.05/ln0.63397 = 6.57. Therefore, the photographer should take 7 pictures.

2 Of the 35 different hexominos, 11 can be folded along edges joining the squares to form a cube. One approach is to try to fold each of the 12 possible pentaminos to form an open-top box. Eight pentaminos can be so folded. Then mark the open-top edges and unfold and add the sixth square to each marked edge to get valid hexominos and discard duplicates. Construct a grid five squares long by three squares high and number the squares in the first row A, B, C, D, E; second row F, G, H, I, J; and third row K, L M, N, O. Then the 11 hexominos are AFGHIK, AFGHIL, AFGHIM, AFGHIN, BFGHIL, BFGHIM, AFGHMN, BFGHMN, CFGHMN, ABCHIJ, and ABGHMN.

3 The solution to ALAS + LASS + NO + MORE = CASH is 1215 + 2155 + 96 + 3684 = 7150. There are 15 solutions. This is the one with the biggest NO. This solution can best be obtained with a simple computer program.

4 Al's final score was 114. Let S be Al's score after the 15 red balls have been sunk. Then, S_{min} = 15 + 14(2) = 43, and S_{max} = 15 + 14(7) = 113. Thus, S must be a semiprime between 43 and 113. The only such values are 46, 49, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 93, 94, 95, 106, and 111. But, the score after the 14th colored ball has been sunk is also a semiprime, so S – 1 must also be a semiprime. This reduces the possibilities for S to 58, 86, 87, 94, and 95. Of these five possibilities, only 87 is never a semiprime when adding successively 2, 3, 4, 5, 6, and 7. Therefore, Al's final score = 87 + 27 = 114. There are a number of sequences that give this score.

5 The sum of the reciprocals of the triangular numbers from 1 to infinity is 2. The nth triangular number has the form $n(n + 1)/2$. Therefore, its reciprocal is $2/[n(n + 1)] = 2[1/n – 1/(n + 1)]$ so that S = 2(1 – ½ + ½ – 1/3 + 1/3 – ¼ + ¼ - 1/5 + …) = 2, since all the terms but the first and last cancel.

Bonus. The expected number of tosses to ultimately get five 6s is 3,698,650,986/283,994,711 = 13.024. The easiest way to solve this problem is by bootstrapping one's way up. Let N_i be the expected number of tosses to get all 6s with i dice.With one die, on the first toss, there is a 1/6 probability of getting a 6 and 5/6 probability of not getting a 6. Thus, we have $N_1 = 1/6 + (5/6)(1 + N_1)$. Solving for N_1 gives $N_1 = 6$. In the following equations, the coefficient of the term involving N_j is given by $C(i, i\text{-}j)(1/6)^{i-j}(5/6)^j$, where $0 \leq j \leq i$, and $C(n, m)$ is the number of combinations of n things taken m at a time. These coefficients are the probabilities of getting i-j 6s out of i dice on the first toss. For N_2, we have $N_2 = 1/36 + (10/36)(1 + N_1) + (25/36)(1 + N_2)$. Solving for N_2 gives $N_2 = 96/11$. Similarly, for N_3 we have $N_3 = 1/216 + (15/216)(1 + N_1) + (75/216)(1 + N_2) + (125/216)(1 + N_3)$ or $N_3 = 10{,}566/1001$. For four dice, we have $N_4 = 1/1296 + (20/1296)(1 + N_1) + (150/1296)(1 + N_2) + (500/1296)(1 + N_3) + (625/1296)(1 + N_4)$ or $N_4 = 728{,}256/61{,}061$. Finally, $N_5 = 1/7776 + (25/7776)(1 + N_1) + (250/7776)(1 + N_2) + (1250/7776)(1 + N_3) + (3125/7776)(1 + N_4) + (3125/7776)(1 + N_5)$ or $N_5 = 3{,}698{,}650{,}986/283{,}994{,}711 = 13.02366$.

Computer Bonus. The smallest weak prime whose reverse is also a weak prime is 3,376,225,859. Both 3,376,225,859 and its reverse 9,585,226,733 are weak primes.

NEW SUMMER PROBLEMS

1 Peter, Quentin, Ralph, Sam, and Thomas went on a photography safari for 10 days. A different pair did the picture taking each day—not always successfully, but the expedition came home with photos of five animals, each snapped on different days. Thomas had no hand in snapping the antelope, but Quentin was one of the pair who got the elephant. No one was on teams that snapped both the antelope and the dromedary, but one proud chap can now brag that he snapped both the crocodile and the elephant. Someone snapped a baboon and also the animal that he snapped while out with Sam. Peter and Quentin had a successful day together, but Ralph and Thomas drew blank. Each man had a hand in photographing two animals, and Quentin, Sam, and Thomas a hand in all five among the three of them. Which pair accounted for each photograph?

—Martin Hollis

2 Find three different integers, P, Q, R, such that P+Q, P+R, Q+R, P-Q, P-R, and Q-R are all squares of integers. The three integers may be positive, negative, or zero. Find the set {P, Q, R} with the smallest P+Q+R.

—Stephen Ainley

3 In a stud poker game with no ante, on the first face-up round, high card made a bet with two coins. Each of the other players called without raising, e.g., made the same bet. Only standard U.S. coins, up to and including silver dollars, were used in the game. Second hand put in three coins; 3rd hand put in two coins and took out one in change; 4th hand put in three coins and took out one in change; 5th (last) hand put in one coin and took out in change all but three of the coins then in the pot. Whenever coins are removed from the pot, they have different values than coins put in the pot. How much did 1st hand bet?

—*Math. Puzzles for Beginners and Enthusiasts*, Geoffrey Mott-Smith

4 Find all integral solutions of $(x+1)^y = x^{(y+1)} + 1$. Assume 0^0 is 1.

—Meyl, 1876

5 Four people each flip a fair coin five times. What is the exact probability that they each get the same number of heads?

—Adapted from *Duelling Idiots and other Probability Puzzlers* by Paul J. Nahin

Bonus. Five beads are strung on a horizontal insulated wire in the shape of a unit square, with the holes in the beads being large enough so that the beads can move to any location on the wire (that is, they can go around the corners of the square).

If each bead is given exactly the same negative electrical charge and the system is allowed to come to equilibrium, what are the coordinates of the five beads (with the non-symetric bead at the origin and an edge of the square on the x-axis)? We want both solutions.

—Daryl Cooper

Computer Bonus. Consider the integer 7. Its cube is 343. The integer divisors of 343 are 1, 7, 49, and 343. Those divisors sum to 400, which is the square of the integer 20. Find the next larger integer such that the sum of the integer divisors of its cube is the square of an integer.

—Fermat, 1657

Postal mail your answers to any or all of the Brain Ticklers to Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697, or email plain text (no HTML, no attachments) to *BrainTicklers@tbp.org*.

The cutoff date for entries to the Summer column is the appearance of the Fall BENT during early October.

The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded.

Jim will forward your entries to the judges, who are: **H.G. McIlvried III,** *PA Γ '53*; **D.A. Dechman,** *TX A '57*; **J.L. Bradshaw,** *TX A '57*; and the columnist for this issue,

—**F.J. Tydeman,** *CA Δ '73*