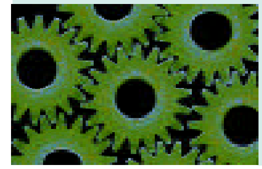
Brain Ticklers



FIFTIETH ANNIVERSARY

The first Brain Tickler in THE BENT appeared in 1951. We continue our observance of the 50th anniversary by repeating below, as the weight-balancing Double Bonus, one of the problems that was used in 1951.

Here are a few statistics from 1961-70, the second decade of our existence: 1,466 entries were submitted during this period, compared with 1,468 for the first 10 years. The most perfect solutions came from William G. Tuel Jr., *NY* Γ *'62;* Robert H.A. Meyer, *NJ* A *'53;* and Martin Crawford, *TN* A *'54.* Submitters with the most correct Bonus Solutions were the same William G. Tuel Jr. and Robert H.A. Meyer, plus Toby Berger, *CT* A *'62.* There were no repeats from the first 10 years.

WINTER REVIEW

Following our practice of making a mistake at least every century, we have done it again. The answer of 16 coins given in the Spring column for the Winter No. 1 problem is incorrect. The problem was, how many hexadecimal coins (of value 1, 5, 10H, 25H, and 50H) must a traveler carry in order to have the least total value of coins and still be able to exactly pay any toll up to 100H? We now say the correct answer is **15 coins,** as reported by John L. Bradshaw, J. Charles Rasbold, and Michael B. Stone. It is achieved with 6 cents, 2 nickels, 5 dimes, no quarters, and 2 halves. For the variation mentioned in the Spring column (to determine the minimal number of coins if there is no restriction on their total value), the answer is 13 coins, for which one of several groupings is 5 cents, 2 nickels, 2 dimes, 2 quarters, and 2 halves.

For good measure, the answer of 13 triangles given in the Spring column for the Winter Bonus problem is also incorrect. This problem was to dissect a square into different-sized similar triangles having a leg ratio of 2 to 1. A better solution of **8 triangles** has been found by Victor J. Gatto and Richard I. Hess. For a square from (0,0) to (10,10), the dissecting lines can be (0,0) to (5,10), (5,10) to (10,0), (6,8) to (10,10), (5,10) to (5,2), (5,2) to (9,2), (9,2) to (9,0), and (9,0) to (1.8,3.6).

We regret these errors and the ensuing embarrassment of the judges.

SPRING ANSWERS

Solutions for the Spring Brain Ticklers are given below. Spring entries will be acknowledged in the Fall issue.

1. The electronic slot-machine problem was to arrange nine digits in a circle such that the numbers formed by clockwise pairs do not have any two-digit prime factors. A check of the integers from 12 to 98 shows that there are 27 with no two-digit prime factors. Examination of these numbers shows that 27 is a good place to start, since it is the only one ending in 7. The next digit must then be 5, and this must be followed by 4 or 6. Continuing in this way with a combination of logic and some trial and error leads to the solution shown.

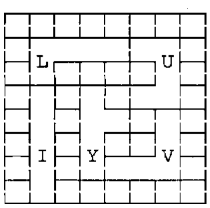
3 2
6 7
1 5
8 4
9

2. TEN x TEN– TEN= NINETY
6 7 4 x 6 7 4 – 6 7 4 = 4 04 7 61
This solution is in base 11. In any base b, there are only $(b\text{-}1)^2(b\text{-}2)$ possible values for TEN, so the amount of trial is limited. However, we can decrease it even further by noting that $(T+1)^2$ must be a two-digit number, and N must equal the first digit of that number or one more than that. We know that b is not 10. Experimenting with b = 5,6,7,8,9,11 yields a solution for b = 11. The next solution happens to be for b = 66.

3. The aviary with various birds has seven eagles. From the problem statement A (auks) must be 16, 81, or 256; B must be 55, 66, or 171; C=13n; A<D = 89 or 233; $(A+C)/(A+B+C+D)$ = 5/8; and $A+B+C+D+E$ = 401. These conditions can be combined to give B = 55; D = 89; C,A = (234,16) or C, A = (169,81); and E = 7.

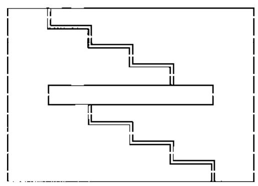
4. Given a random nine-digit number containing all digits 1 to 9, the probability of it being divisible by 11 is 11/126. A number is divisible by 11 if the sum of the even-position digits minus the sum of the odd-position digits is divisible by 11. The four even-position digits can be chosen in $C(9,4) = 126$ ways. Let S_E equal the sum of the even-position digits. Then $10 \le S_E \le 30$. Since the sum of the digits from 1 through 9 is 45, $S_O = 45 - S_E$ and $S_O - S_E = 45 - 2S_E$. Therefore, if $45 - 2S_E$ is divisible by 11, then the nine-digit number will also be divisible by 11. The minimal and maximal values of $45 - 2S_E$ are –15 and 25. Within this range, only –11, 0, 11, and 22 are divisible by 11, and only –11 and 11 lead to an integral value for S_E, namely 28 or 17. It is easy to list the 11 possible four-digit combinations that sum to 17 or 28. For each of these there are 4!5! possible nine-digit numbers. Therefore, $P = 11(4!5!)/9! = 11/126 = 0.0873$.

5. Five is the minimum of pentominoes from a complete set that can be placed on a chessboard so that no other pentomino from the set of 12 can be placed. Several arrangements are possible; one is shown in the figure, where the letters refer to the "names" for the pentominoes. The trick is to pick the pentominoes that are the most difficult to block, i.e., the ones that are most linear, and use these to block the rest.



Bonus. You were to find the envelope of coverage for an antiaircraft gun with muzzle velocity v. Consider a coordinate system with origin at the gun, x horizontal and y vertical. The velocity equations for the shell are $dx/dt = v \cos\theta$ and $dy/dt = v \sin\theta - gt$, where θ is the gun's elevation angle. Integration gives equations for the trajectory: $x = vt \cos\theta$ and $y_T = vt \sin\theta - gt^2/2$. Elimination of t leads to $y_T = x \tan\theta - x^2/(4h \cos^2\theta)$, where $h = v^2/(2g)$ is the maximal height of a shell fired at $\theta = \pi/2$ and $2h$ is the maximal horizontal range of a shell (which occurs for $\theta = \pi/4$). The maximal y_T for a given x defines a point on the envelope curve; $dy_T/d\theta = 0$ occurs when $\tan\theta = 2h/x$ which then gives $y_E = h - x^2/(4h)$. The desired envelope E, or volume, is the integral from 0 to $2h$ of $2\pi x y_E dx$. The integration result is $E = 2\pi h^3 = \pi v^6/(4g^3)$.

Double Bonus. The 9 x 12 rug with a 1 x 8 hole can be cut as shown so that the two pieces can form a 10 x 10 rug.



NEW PROBLEMS

1. They were one of those Tau Beta Pi bi-spousal couples. One day, she went on an errand. That evening, he asked her how long the errand had taken. Knowing he also liked puzzles, she responded, "When I left, the hands on my watch were aligned, with the hour hand between 8 and 9 and the minute hand between 2 and 3. When I returned, the hands were again aligned, but this time the hour hand was between 2 and 3, and the minute hand was between 8 and 9. You tell me how long I was gone." To the nearest second, how long did the errand take?

—Justo T. Del Puerto

2. A palindrome is a number that reads the same forward and backward. How many integers in the range from 1 through 10^{10} are palindromic?

—**Edward C. Creutz,** *PA* Γ *'36*, and
Michael J. Creutz, *CA* B *'66*

3. Solve the following cryptic multiplication, where FLY is exactly divisible by GO. GO x FLY = KITES

—**Don A. Dechman,** *TX* A *'57*

4. On the first day of spring on the level plains of Ecuador, a certain tribe of tall Indians celebrates the fact that they can see the sun for a longer time than a neighboring tribe of short Indians. In fact, how much longer, in seconds, is the time between sunrise and sunset for the tall tribe (whose eyes are 200 cm above the ground) as compared to the short tribe (whose eyes are 140 cm above the ground)?

—**Byron R. Adams,** *TX* A *'58*

5. In order to help deal with the changing postage rates, an efficiency expert tried to design a 2 x 3 pane of stamps in such a way that by using a single stamp or a connected set of them, all possible postage rates 1, 2, 3, …, N could be met exactly. The first attempt yielded the arrangement below, which can satisfy the values from 1¢ to 32¢ except for 18¢; therefore, N = 17 for this arrangement. The maximum would be 63, of course (1+2+4+8+16+32), but there is no way to reach that total while meeting the requirement for all sets of stamps to be contiguous. Find the largest N and show its 2 x 3 array.

10	9	7
2	3	1

—Adapted from *New Scientist*

Bonus. Riley has ten spherical beads, 1 cm in diameter. Two are red, two are orange, two are yellow, two are green, and two are blue. The beads are to be strung to make a circular bracelet. How many different bracelets can be formed?

—**Howard G. McIlvried III,** *PA* Γ *'53*

Double Bonus. A chemist had twelve 10-gram weights, one of which he knew was inaccurate. By conducting only three weighings on his equal-arm balance, he determined which weight was inaccurate and whether it was too light or too heavy. How did he do it?

—**Maurice W. Widener,** *TX* A *'48*

The judges are: H.G. McIlvried III, F.J. Tydeman, D.A. Dechman, and the columnist for this issue:

—**R. Wilson Rowland,** *MD* B *'51.*