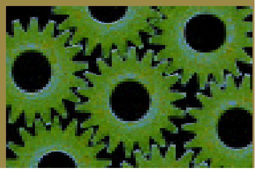


Brain Ticklers

RESULTS FROM FALL 2003

Perfect

* Baines, Elliot A., Jr.	NY	Δ	'78
Christenson, Ryan C.	UT	B	'92
* Creutz, Michael J.	CA	B	'66
Creutz, Edward C.	PA	Γ	'36
* Garnett, James M.	MS	A	'65
Griggs, James L., Jr.	OH	A	'56
* Kimsey, David B.	AL	A	'71
Peterson, David L.	MA	A	'79
Rentz, Peter E.	IN	A	'55
* Snelling, William E.	GA	A	'79
* Stribling, Jeffrey R.	CA	A	'92
Strong, Michael D.	PA	A	'84
Weinstein, Stephen A.	NY	Γ	'96

Other

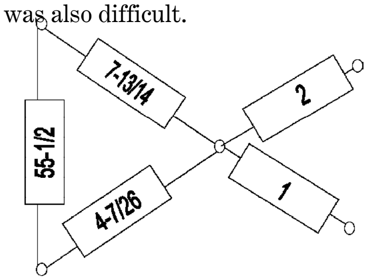
Alexander, Jay A.	IL	Γ	'86
Anderson, Kurt E.	KS	Γ	'90
Anidi, Chibueze E.	Non-member		
Aron, Gert	IA	B	'59
Atobatele, Timothy A.	NV	B	'02
Barnas, Neil B.	NM	Γ	'01
* Barthel, Gerald R.	OH	B	'67
Beam, Charles P.	SC	A	'92
Biggadike, Robert H.	AR	A	'58
Brule, John D.	MI	B	'49
Brzezinski, Mark A.	OH	B	'00
Chipps, Jason F.	WV	A	'95
Conway, David B.	TX	I	'79
Couillard, J. Gregory	IL	A	'89
Eckley, Paul L.	NV	A	'75
Fine, Joseph M.	PA	Z	'75
Gallen, Kevin B.	PA	Θ	'00
Garside, Jeffrey J.	WI	E	'90
* Gifford, William C.	AR	A	'64
Ginat, Daniel T.	NY	A	'01
Ingley, John S.	NY	Δ	'59
Jones, Donlan F.	CA	Z	'52
Lalinsky, Mark A.	MI	Γ	'77
Lesser, Lawrence M.	Son of member		
Lew, Thomas M.	TX	Δ	'84
Londot, Keith L.	OH	Γ	'76
Marks, Lawrence B.	NY	I	'81
Marrone, James D.	IN	A	'87
Nabutovsky, Joseph	Father of member		
* O'Donnell, Craig R.	PA	A	'03
Penkala, Stanley J.	PA	Δ	'65
Quintana, Juan S.	OH	Θ	'62
Rasbold, J. Charles	OH	A	'83
Reid, Barry G.	PA	Z	'74
Robinson, Alec M.	MI	Γ	'88
* Routh, Andre G.	FL	B	'89
Schlak, Robert P.	TX	B	'91
* Schmidt, V. Hugo	WA	B	'51
Snyder, M. Duane	IA	B	'63
* Spong, Robert N.	UT	A	'58
Sprinkle, Jonathan M.	TN	Γ	'99
Sumariwalla, Zubin M.	MO	B	'94
Summerfield, Stuart	Son of member		
* Thaller, David B.	MA	B	'93
* Tom, Danny T.	CA	A	'99
Valko, Andrew G.	PA	Λ	'80
* Voellinger, Edward J.	Non-member		
Wechsler, Lawrence D.	NY	E	'55
* Yee, David G.	NJ	B	'04

* Denotes correct bonus solution.

A perfect Summer entry from James L. Griggs Jr., *OH* A *'56*, was inadvertently omitted from the entry list in the Winter issue.

FALL REVIEW

The most difficult Fall regular problem was No. 1 about Mycroft's telegram. No. 4, about how fast the earth must spin to overcome gravity, was also difficult.



Several responders sent in a solution to the Fall Bonus different from the one we provided. In the figure, the numbers represent resistances in ohms. This approach, however, uses many more 1-ohm resistors than the solution we provided. Note that a resistance of m/n ohms can be achieved by wiring n 1-ohm resistors in parallel to give a resistance of $1/n$ ohms and then wiring m of these units in series. In addition, Joseph M. Fine, *PA* Z *'73*, sent a solution with six terminals that provides all the resistances from 1 ohm to 15 ohms.

Stephen A. Weinstein, *NY* Γ *'96*, has provided a clever solution to the Fall Computer Bonus that doesn't require the use of a computer; $(10N)!$ equals the product of $2N$ factors divisible by 5 times $4N$ even factors not divisible by 5 times $4N$ odd factors not divisible by 5. The product of the factors divisible by 5 is $5^{2N}(2N)!$. The product of the $4N$ even factors not divisible by 5 equals 2^{4N} times an integer ending in 4 if N is odd or in 6 if N is even. The product of the $4N$ odd factors not divisible by 5 is an integer ending in 9 if N is odd or in 1 if N is even. Thus, $(10N)!$ equals $5^{2N}2^{4N}(2N)!$ times an integer ending in 6, but $5^{2N}2^{4N} = 10^{2N}4^{N}$, which is a number whose last nonzero digit is 4 if N is odd or 6 if N is even. Therefore, the last nonzero digit of $(10N)!$ equals the last nonzero digit of $Z(2N)!$, where $Z = 4$ if N is odd and $Z = 6$ if N is even. If $2N$ is not divisible by 10, then $2N = 10M + X$, and $(2N)! = (10M + X)(10M + X - 1)\ldots(10M + 1)(10M)!$ By repeatedly applying these relationships, one can fairly quickly show that the last nonzero digit of 1,000,000! is 4.

WINTER SOLUTIONS

Although the entries for the Winter Ticklers will not be acknowledged until the Summer column, here are the answers.

1. The largest integer such that each pair of consecutive digits is a different prime is 619,737,131,179. There are 21 two-digit primes. However, eleven of these start with an even digit or a five and can, therefore, be only at the beginning of the number. This leaves ten primes (11, 13, 17, 19, 31, 37, 71, 73, 79, 97) that can occur elsewhere in the number. Obviously, the best we can do is to use all ten of these primes to form an eleven-digit number. A little trial and error quickly shows that 19,737,131,179 is the largest number that can be formed using all ten primes. The largest of the remaining two-digit primes that ends in 1 is 61, which, when appended to the front of this number, leads to the answer above.

2. DOS × DOS = CUATRO (2 × 2 = 4, in Spanish) has the unique solution, 564 × 564 = 318,096. DOS must be between 352 and 987 to generate a six-different-digits square. Thus, D can be only 3, 4, 5, 6, 7, 8, or 9. Also, it is clear that S can be only 2, 3, 4, 7, 8, or 9, with corresponding values for O of 4, 9, 6, 9, 4, and 1. This limits DOS to a few values that can quickly be checked to arrive at the above answer.

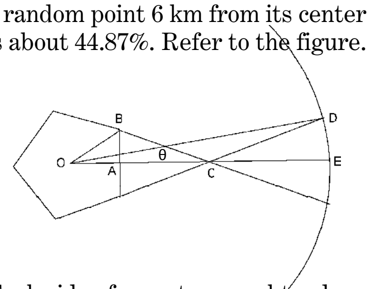
3. Given a series of five real numbers such that $n_{i+1}/n_i = Q$, then $n_1 + n_2 + n_3 = n_4 + n_5$ only for values of Q of approximately – 1.5128764 and 1.1787242. Dividing the above equation by n_1 and realizing that $n_{i+1}/n_1 = Q^i$ gives, after rearranging, $Q^4 + Q^3 - Q^2 - Q - 1 = 0$. This is a quartic equation, and there is a formula for its solution. However, the analytic approach is quite complicated, and it is easier to use trial and error to find the above answer. The equation also has two complex roots.

4. The only 4 × 4 grid in which each column and each row is a perfect square with no zero digits is:

2	1	1	6
1	2	2	5
1	2	9	6
6	5	6	1

Since all squares of integers (with no 0 digits) end in 1, 4, 5, 6, or 9, these are the only digits that can appear in the last row and last column. The only four-digit squares containing only these digits are 1,156, 1,444, and 6,561. Since the last digits of these numbers are all different, it is clear that the last row and the last column must be the same. There are only a few possibilities for the rest of the rows and columns for each of these possibilities. Trying these shows that the above solution is unique.

5. The probability of being able to see three sides of the Pentagon from a random point 6 km from its center is about 44.87%. Refer to the figure.



Each side of a pentagon subtends an angle of 72°; therefore, AOB = 72°/2 = 36°, OBA = 90° – 36° = 54°, ABC = 180° – 2(54°) = 72°, OCB = 90° – 72° = 18°, and OCD = 180° – 18° = 162°. Since AB = 140.5 m (half the length of the side of the Pentagon), OA = 140.5tan54° and AC = 140.5tan72°. Therefore, OC = OA + AC = 140.5(tan54° + tan72°) = 625.796 m. Let COD = θ; then OCD = 18° – θ. Now, by the law of sines applied to ΔOCD,

	Standard Dice		One Loaded 5-face	
Initial Throw	Prob. of Throw (P_t)	Prob. of Winning (P_w)	Prob. of Throw (P_t)	Prob. of Winning (P_w)
2	1/36	0	1/45	0
3	1/18	0	2/45	0
4	1/12	1/3	1/15	2/7
5	1/9	2/5	4/45	8/23
6	5/36	5/11	13/90	13/28
7	1/6	1	1/6	1
8	5/36	5/11	13/90	13/28
9	1/9	2/5	11/90	11/26
10	1/12	1/3	1/10	3/8
11	1/18	1	7/90	1
12	1/36	0	1/45	0
$\Sigma P_t P_w$	244/495 = 0.4929		780,223/1,506,960 = 0.5178	

6,000/sin162° = 625.796/sin(18° – θ). Therefore, sin(18° – θ) = 0.03223, which gives 18° – θ = 1.84698, or θ = 16.153°. Thus, because arc DE is one tenth of the length from which three sides of the Pentagon can be seen, the desired probability is 10(16.153)/360 = 0.4487.

Bonus. If the unethical gambler loads the 5-face of one die, he will increase his odds of winning from 244/495 to 780,223/1,506,960 (from about 49.293% to 51.775%, that is, from slightly less than 50% to a little over 50%). Intuitively, 5 looks like the best number to load, since it is the only value that can form both a 7 and an 11 but not a 2, 3, or 12. The above table shows how loading the 5-face on one die (so that the probability of a 5 is 1/3 and the probabilities of other values is 2/15) changes the probability of winning.

The overall probability of winning is calculated by multiplying the probability of throwing a given number by the probability of winning given that that number is thrown. A similar table, generated for loading other faces, gives P_1 = 0.4618, P_2 = 0.4844, P_3 = P_4 = 0.5052, and P_6 = 0.4951. Thus, loading the 5-face is best for the gambler.

Computer Bonus. A maxdigital number is a ten-digit integer that contains the digits 0 through 9 each exactly once. The largest maxdigital numbers that are 3, 6, 7, or 9 times other maxdigital numbers are:

9,875,304,162 = 3 × 3,291,768,054
9,875,034,162 = 6 × 1,645,839,027
9,867,430,125 = 7 × 1,409,632,875
9,876,314,205 = 9 × 1,097,368,245

NEW SPRING PROBLEMS

Here are some new Ticklers to keep you occupied during Spring break. Replies to these problems will be acknowledged in the Fall column.

1. Lance wants to prepare a set of building blocks, consisting of wooden cubes, for his daughter. He decides to paint all the faces of all the cubes, each face with a single color with no two adjacent faces having the same color. As he starts work, he discovers that the only colors of paint he has are red, orange, yellow, green, and blue. If no two blocks are to be exactly alike, what is the maximum number of blocks that can be in the set?

—*nearly impossible Brain Bafflers* by Tim Sole and Rod Marshall

2. Solve the following four simultaneous equations:

$$x = 25 - 24\cos A$$
$$x = 34 - 30\cos B$$
$$x = 41 - 40\cos C$$
$$A + B + C = 2\pi$$

All angles are positive and measured in radians. Express x as an algebraic number, that is, as an expression involving only rational numbers and surds of rational numbers.

—Joseph S. Brock

3. After four rounds of the golf tournament, Ann, Betty, Carol, and Donna were tied, even though the individual scores for the 16 rounds they played were all different. The 16 scores were all in the 60s and 70s. Each of Ann's four scores was a prime. Betty's four scores were each a semiprime (the product of two primes). None of Carol's or Donna's scores was either prime or semiprime. Carol's lowest round was better (that is, lower) than Donna's lowest round, and Carol's worst round was better than Donna's worst round. What were the scores of each of the golfers for the four rounds?

—Richard England in *New Scientist*

4. If m points on one parallel line are joined with straight-line segments to n points on a second parallel line in all possible ways, what is the maximum number of points of intersection in the area between the two parallel lines, not counting the original m plus n points?

—George Chrystal (1851-1911)

5. Lou, Mac, and Neil were having lunch with their friend Zoey, when they began reminiscing about the past soccer season. Neil remarked, "Among the three of us, we scored PQ goals last season." "How many did you score?" asked Zoey. "All but RS of them," Neil replied. "Lou scored QR (which is all but QP of the total), and Mac scored SR." If P, Q, R, and S each represent a different nonzero decimal digit, how many goals did each man score? Note that PQ represents a two digit number, not the product of two digits.

—*New Scientist*

BONUS. We are interested in finding a sequence of $2n + 1$ consecutive positive integers, such that the sum of the squares of the first $n + 1$ integers equals the sum of the squares of the last n integers. The simplest such sequence is $3^2 + 4^2 = 5^2$. Give a formula in terms of n for the first integer in such a sequence.

—*The Numerology of Dr. Matrix* by Martin Gardner

COMPUTER BONUS. Find a 4×4 magic square in which all the elements are different prime integers less than 100. In a magic square, the sum of every row, every column, and the main diagonals are each the same number, known as the magic sum. Remember, 1 is not a prime number. There are many such magic squares, but we want one that has a magic sum of 202.

—*Numbers: Fun & Facts* by J. Newton Friend

Send your answers to any or all the Spring Ticklers to: Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville TN 37901-2697.

The cutoff date for submitting answers is the appearance of the Summer issue in July.

If your answers involve text only, they may be emailed to *Brainticklers@tbp.org.* The details of your answers are not needed, unless you think you have an unusual approach which would be of interest to the judges.

The Computer Bonus is not graded. If you have a favorite problem of your own, feel free to include it. Jim will forward your entries to the judging panel, consisting of **F.J. Tydeman,** *CA* Δ *'73*, **D.A. Dechman,** *TX* Α *'57*, **J. L. Bradshaw,** *PA* Α *'82*, and the columnist for this issue, **H.G. McIlvried III,** *PA* Γ *'53*.

HEADQUARTERS VISITORS

Robert O. Barr Jr., P.E., *Michigan Gamma '61*, Okemos, MI; November 14, 2002.
Alberta Barr, Okemos, MI; November 14, 2002.
Edward D. Basta, *Ohio Epsilon '82*, Chesterland, OH; February 26, 2003.
J.P. Blackford, *District of Columbia Gamma '95*, Washington, DC; March 21, 2003.
George T. Laughlin II, *Colorado Delta '75*, Fort Collins, CO; April 9, 2003.
Gregory T. Schroeder, *Michigan Epsilon '00*, Detroit, MI; May 2, 2003.
Dawn Schroeder, Detroit, MI; May 2, 2003.
Craig A. Elder, *Michigan Epsilon '00*, Detroit, MI; May 2, 2003.
Goran Jancevski, *Michigan Epsilon '01*, Detroit, MI; May 2, 2003.
Kristine M. Patterson, *Michigan Epsilon '00*, Detroit, MI; May 2, 2003.
Jennifer R. Peters, *Michigan Epsilon '04*, Detroit, MI; May 2, 2003.
Curtis D. Gomulinski, *Michigan Epsilon '01*, Detroit, MI; May 2, 2003.
Sherry Jennings-King, *Tennessee Alpha '93*, Plymouth, MN; July 3, 2003.
Gregory King, Plymouth, MN; July 3, 2003.
Adedeji B. Badiru, *Tennessee Gamma '79*, Knoxville, TN; July 16, 2003.
Matthew W. Ohland, *Florida Alpha '96*, Clemson, SC; August 18, 2003.
Ellen D. Styles, *Alabama Delta '85*, Huntsville, AL; August 22, 2003.
Way Kuo, P.E., *Iowa Alpha '79*, November 13, 2003.
Louis M. Clark, *Tennessee Alpha '94*, Atlanta, GA; November 20, 2003.
Norman P. Pih, *Tennessee Alpha '82*, Flagstaff, AZ; November 22, 2003.
Michael T. Abraham, Austin, TX; December 10, 2003.
Clive Frazier, *Wisconsin Alpha '66*, Orlando, FL; December 30, 2003.
Russell W. Pierce, *Washington Alpha '70*, Puyallup, WA; January 23, 2004.
David E. Farrow, *Texas Eta '89*, Martin, TN; February 6, 2004.