FIFTIETH ANNIVERSARY

This issue marks the fiftieth anniversary of the appearance of the first Brain Tickler in the April 1951 issue of The Bent. That first problem, the famous rug problem, was repeated in the July 1951 issue to give more people a chance to respond. Three new problems, under the name Brain Ticklers, appeared in the December 1951 issue, but no solutions or acknowledgments were published. A judging panel was then assembled, and regular columns have appeared since April 1952.

The first Bonus problem appeared in the December 1956 issue, the first Double Bonus appeared in April 1966, and the column assumed the general format it has maintained ever since, with the Double Bonus sometimes replaced by a Computer Bonus. The rug puzzle is repeated as the Double Bonus in this column. The three December 1951 Ticklers will also be repeated in later columns.

During the next five columns, we will present some statistics from each of the five decades of our existence. During 1951-60, 1,468 entries were submitted. During this decade, the most perfect sets of solutions to the regular problems were received from Lawrence Israel, *NY* Λ *'56*, Robert L. Patton, *TX* Δ *'33*, and P. P. Gillis, *RI* A *'53*. And the most correct bonus solutions were received from Lawrence Israel, *NY* Λ *'56*, Howard G. McIlvried III, *PA* A *'53*, and Byron R. Adams, *TX* A *'58*. Four of these top five solvers in our first decade became Brain Tickler judges.

FALL REVIEW

The toughest regular Fall Tickler was No. 3 about the LED clock, with many responders giving the answer for the greatest absolute change in brightness, rather than the greatest relative change, as asked for in the problem.

Incidentally, a neat trial-and-error solution to the Fall Double Bonus about triangle ABC involves using two pins, a loop of string, and a pencil. Stick the pins in points A and B, loop the string around them, and draw an ellipse by holding the pencil against the string and moving it around. Repeat for points A and C, and points B and C. The point of intersection of these three ellipses is the point where triangles ABP, ACP, and BCP have equal perimeters. If the three ellipses don't meet in a point, adjust the size of the loop and try again.

WINTER ANSWERS

Those who responded to the Winter Ticklers will be acknowledged in the Summer column. In the meantime, here are the answers.

1 The problem was, how many hexadecimal coins (of value 1, 5, 10H, 25H, and 50H) must a traveler carry in order to have the least total value of coins and still be able to exactly pay any toll up to 100H? The correct answer is **15 coins,** and it is achieved with 6 cents, 2 nickels, 5 dimes, no quarters, and 2 halves.

An interesting variation on this problem is to determine the minimal number of coins needed, if there is no restriction on their total value. The answer is 13 coins, for which one of several groupings is 5 cents, 2 nickels, 2 dimes, 2 quarters, and 2 halves.

2 The best solution we have is that 12 knights are required so that every square on a standard chessboard is either occupied or threatened. Label the columns 1 through 8 and the rows A through H. Place knights on squares C2, C3, and D3. These three knights occupy or threaten all the squares in the upper left quadrant of the chessboard except B3, C4, and D2. However, we note that if the pattern in the upper left quadrant is repeated in the upper right, lower right, and lower left quadrants, except that the pattern is rotated 90° clockwise upon each repetition, then all 64 squares will either be occupied or threatened. Therefore, the other nine knights are located at B6, C5, C6, E6, F6, F7, F3, F4, and G3. The mirror image of this solution also works.

3 The number 35,964 has the curious property that it is equal to the sum of all the three-digit permutations of its five digits. Let the number we are seeking be $ABCDE$. Then the number of three-digit permutations is $P(5, 3) = 5!/(5 - 3)! = 60$, which is the number of different arrangements of three digits chosen from the five digits in the number. Since there are five digits, each digit must appear in the hundreds, tens, and units places 12 times. Therefore, $(1{,}200 + 120 + 12)S = 1{,}332S$, where $S = A + B + C + D + E$; but S must be between 15 and 35, inclusive. Trying all possibilities, we find that only $S = 27$ gives a product, the sum of whose digits equals S, i.e., $1{,}332(27) = 35{,}964$, and $3 + 5 + 9 + 6 + 4 = 27$.

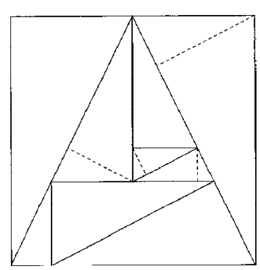
4 For there to be just less than a 50% chance that Zero was the culprit, 168 people would have had to remain in town with him. Let Pn be the probability that Zero is the culprit, where n is the number of people in town, and let $P_{0,n}$ be the probability that there are no 00X plates in this group, $P_{1,n}$ the probability of one 00X plate, $P_{2,n}$ the probability of two 00X plates, etc. Then we need to find n such that

$P_n = P_{0,n} + P_{1,n}/2 + P_{2,n}/3 + \ldots + P_{9,n}/10$ is just under 0.5. Now, since there are 999 vehicles (not counting Zero's), and 990 are without 00X plates, $P_{i,n} = C(9, i)C(990, n - i)/C(999, n)$, where $C(m, n)$ is the number of combinations of m things taken n at a time. $P_{0,n}$ can be calculated by multiplying (990/999)(989/998)(988/997). . . for n terms (a spreadsheet helps). Once $P_{0,n}$ has been calculated, the rest of the $P_{i,n}$s can be calculated from $P_{i,n} = [C(9, i)/C(9, i - 1)]$ $[(n - i + 1)/(990 - n + i)]P_{i-1,n}$. Applying these equations with various values of n, we find that $P_{167} = 0.50149$, and $P_{168} = 0.49964$. Therefore, 168 citizens must remain in town for P_n to be just slightly less than 50%.

5 The digits I started with in the product table were 3, 5, 6, and 7. The first digit must be 2 or 3, since it has a single digit square different from itself. If it is 2, then its square is 4, and the only possible sequences are 2567 or 2568. But neither of these work, since the product of the third and fourth digits must be a two-digit integer, whose digits are both different from any in the sequence. Therefore, the first digit of the sequence must be 3. The third digit cannot be 5, because then the second digit must be 4, and the first digit of 5 times 6, 7, or 8 is 3 or 4. Therefore, the third and fourth digits are 67, 68, or 78; but 6 x 8 = 48 and 7 x 8 = 56. Thus, the third and fourth digits are 67, which makes the second digit 5.

Bonus. One's first impulse is to divide the square into four congruent right triangles and then further divide these triangles by dropping perpendiculars from the right angles to the hypotenuses, but trying to divide the fourth triangle quickly leads to endless proliferation of triangles. Thus, one has to seek an arrangement with no more than three congruent triangles. Start by drawing lines from the bottom corners of the square to the center of the top, as shown in the accompanying figure. This divides the square into two outer congruent triangles and a central triangle. Divide one of the outer triangles into two smaller triangles. Divide the central triangle into a trapezoid and a triangle by drawing a line parallel to its base and a third of the way up. Divide the triangle in half by dropping an altitude, and divide the trapezoid into three triangles, as shown. This will leave you with three congruent triangles. Divide one of these into two smaller triangles and one into five smaller triangles. This divides the square into 13 triangles with sides in a ratio of 2 to 1.

A simple calculation will show that they all have different areas.



A better solution of **8 triangles** has been found by Victor J. Gatto and Richard I. Hess. For a square from (0,0) to (10,10), the dissecting lines can be (0,0) to (5,10), (5,10) to (10,0), (6,8) to (10,10), (5,10) to (5,2), (5,2) to (9,2), (9,2) to (9,0), and (9,0) to (1.8,3.6).

Computer Bonus. 98,876,532,401 is the largest eleven-digit prime that contains each of the digits 0 through 9 at least once. It is clear that the repeated digit cannot be a 9, because then the sum of the digits would be 54 and the number would be divisible by 9. Therefore, starting with 98,876,543,201, test progressively smaller permutations of the listed digits until a prime is found. Obviously, it is not necessary to test even numbers or numbers ending in 5.

NEW PROBLEMS

Now for some stimulating problems to get the next half-century off to a good start.

1 A local eatery has an electronic "slot machine" that offers various games. In one of them, called PrimeTime, after a coin is inserted, the digits 1 through 9 appear in random order around a circle. Then, an arrow, like the minute hand on a clock, spins and stops, pointing to the space between two of the digits. If the two-digit number formed in a clockwise direction by the two digits on either side of the space the arrow points to has a two-digit prime factor, you win the jackpot. For example, if the arrow stopped between 9 and 2, you would win because 92 has the prime factor 23. On the other hand, if your number were 81, you would lose. I recently played the game; and, as the arrow was spinning, I realized to my dismay that no matter where the arrow stopped, I would lose. Starting with 1, what was the clockwise order of the digits?

— Susan Denham in *New Scientist*

2 Don't let the wording of this cryptic fool you into thinking it is solvable in base 10. It isn't.

TEN x TEN - TEN = NINETY

Your job is to solve it in the smallest base for which a solution exists.

— *Journal of Recreational Mathematics*

3 I recently visited a small aviary and asked the keeper how many birds he had. He replied, "I have a total of 401 birds of five different species. The number of auks is a fourth power, although they will be breeding soon. I have more than ten boobies, and their number is both a palindrome and triangular. The number of cuckoos is divisible by 13. The number of doves, which is more than the number of auks, is prime and a Fibonacci number. More than five-eighths of the four species I have mentioned so far are auks and cuckoos. How many eagles do I have?" (A triangular number has the form $n(n + 1)/2$, where n is a positive integer. Such numbers can be arranged in triangles, like bowling pins.)

— *The Platinic Corner*

4 An urn contains nine balls labeled 1 through 9. Draw out the balls one at a time and line them in the order drawn to form a nine-digit number. What is the probability that the number so formed is divisible by 11?

— *Technology Review*

5 A pentomino is a plane figure constructed from five unit squares joined along their edges. A complete set of pentominoes consists of one of each of the 12 different such figures. Rotations and reflections are allowed but do not count as different figures. Starting with a complete set whose squares are the size of chessboard squares, what is the smallest number of pentominoes that can be placed on a chessboard so that none of the remaining pentominoes can be placed on the board? Pentominoes must be placed so that they are congruent with the squares of the chessboard, and they cannot overlap. To see what a complete set of pentominoes looks like, see the Brain Ticklers column in the Fall 1999 issue of The Bent. We like to assign the letter names F, I, L, N, P, T, U, V, W, X, Y, and Z, which the pentominoes at least somewhat resemble.

— *Mathematics Teacher*

Bonus. An anti-aircraft gun with muzzle velocity v can be fired at any azimuth or elevation by vertical and horizontal rotation about its fixed location. What is the total volume of sky that can be reached by a shell from the gun, i.e., what is the envelope of coverage? Assume the land in the vicinity of the gun is level, and ignore wind and air resistance. Express your answer as a function of v and the gravitational constant g.

— **Richard W. de Nobel,** *OH* Α *'52*

Double Bonus. A 9-ft. by 12-ft. rug has a centered 1-ft. by 8-ft. hole, with the 8-ft. dimension parallel to the 12-ft. side. Show how to cut the rug into two pieces that can be fit together to form a 10-ft. by 10-ft. rug with no hole.

The judges are,
F.J. Tydeman, *CA* Δ *'73;*
R.W. Rowland, *MD* Β *'51;*
D.A. Dechman, *TX* Α *'57;* and the columnist for this issue,
Howard G. McIlvried III, *PA* Γ *'53.*