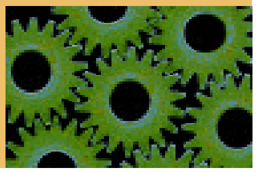


Brain Ticklers

RESULTS FROM SPRING 2004

Perfect

*Alexander, Jay A.	IL	Γ	'86
*Anderson, Paul MacLean	Son of member		
*Argintar, Donald	NY	Γ	'83
*Baines, Elliot "Chip" A., Jr.	NY	Δ	'78
*Biggadike, Robert H.	AR	A	'58
*Celestino, James R.	NJ	B	'00
*Christenson, Ryan C.	UT	B	'93
*Couillard, J. Gregory	IL	A	'89
*Creutz, Michael J.	CA	B	'66
Creutz, Edward C.	PA	Γ	'36
*De Vincentis, Joseph W.	TX	Γ	'93
*Duker, Rachayl Novoseller	NY	A	'97
*Garnett, James M.	MS	A	'65
*Kimsey, David B.	AL	A	'71
*Mayer, Michael A.	IL	A	'89
*Pickelmann, Paul R.	Son of member		
*Quintana, Juan S.	OH	Θ	'62
*Schmidt, V. Hugo	WA	B	'51
*Spong, Robert N.	UT	A	'58
*Stoll, Eric D.	NY	H	'61
*Stribling, Jeffrey R.	CA	A	'92
*Strong, Michael D.	PA	A	'84
*Tom, Danny T.	CA	A	'99
*Verkuilen, William W.	WI	B	'92
*Voellinger, Edward J.	Non-member		
*Weiss, Eric H.	CA	Γ	'98

Other

*Aron, Gert	IA	B	'59
*Bickford, M. Dudley	NH	A	'59
*Bohdan, Timothy E.	IN	Γ	'85
*Bolognese, Christopher A.	OH	Γ	'03
*Brule, John D.	MI	B	'49
*Carver, Robert M.	OH	K	'87
*Conway, David B.	TX	I	'79
*Doniger, Kenneth J.	CA	A	'77
*Fenstermacher, T. Edward	MD	B	'80
*Ford, Robert A.	CA	K	'94
Fulara, Steven P.	IL	Z	'98
*Galer, Craig K.	MI	A	'77
*Griggs, James L., Jr.	OH	A	'56
*Harpole, George M.	CA	E	'74
*Jones, Donlan F.	CA	Z	'52
*Koehn, Laurence J.	OK	Γ	'93
Larson, M. Rhett	KS	Γ	'04
*Lints, Michael C.	NY	Π	'79
Marrone, James I.	IN	A	'61
Mitchell, Gregory K.	NM	B	'87
*Nabutovsky, Joseph	Father of member		
*Peterson, David L.	MA	A	'79
*Post, Irving G.	PA	E	'58
*Powell, Scott F.	NC	Γ	'97
Rasbold, J. Charles	OH	A	'83
Rentz, Peter E.	IN	A	'55
*Shambaugh, C. Sue	MD	A	'85
*Short, Daniel L.	NY	N	'73
Switzer, Geroge F.	CA	M	'85
Tessier, Thomas M.	MA	A	'90
*Thaller, David B.	MA	B	'93
Valko, Andrew G.	PA	Λ	'80
*Van Sickel, James R., Jr.	OH	H	'77
*Vogt, Jack C.	OH	E	'56
Wolff, Nicholas L.	NE	A	'00
*Wolfgang, Robert J.	PA	B	'85
*Yee, David G.	NJ	B	'04

* denotes correct Bonus answer

SPRING REVIEW

Spring No. 1, about painting blocks, turned out to be by far the most difficult problem. Only about half the entries were correct. Spring No. 2, about solving trigonometric equations, had two answers. We gave full credit if either answer was submitted. Allen Klinger provided an elegant solution to the Spring Bonus about sequences of $2n + 1$ consecutive integers in which the squares of the first n+1 integers equal the sum of the squares of the last n integers. His solution is :

> Let a be the middle value of the $2n + 1$ integers. Then, $\Sigma_{i=1}^{n}(a-i)^2 + a^2 = \Sigma_{i=1}^{n}(a+i)^2$. Expanding and combining terms, we get $a^2 = 4a\Sigma_{i=1}^{n} i = 2an(n+1)$. Therefore, $a = 2n(n + 1)$, and the first term of the sequence equals $a - n = 2n^2 + n$.

SUMMER SOLUTIONS

Readers' entries for the Summer problems will be acknowledged in the Winter BENT. Meanwhile, here are the answers:

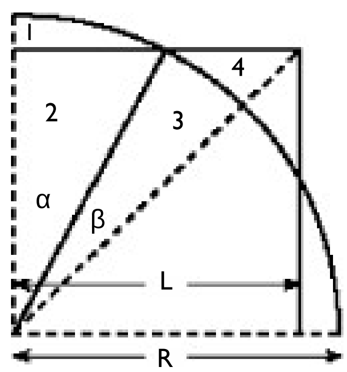
1

		*	*	7	*
			*	7	*
	*	*	*	*	*
*	*	*	2	*	
8	*	5	*		
*	*	*	*	*	*

is solved by 1,475 x 677. Call the terms AB7D and E7F. D must be 5 to satisfy the 2 in ***2*. Then E must be 6 to satisfy the 5 in 8*5*. Then A must be 1 to satisfy the 8 in 8*5*. And B must be either 3 or 4 to also satisfy the 8 in 8*5*. But only B = 4 yields the required five-digit product of 1B75 times 7. F must also be 7 to keep the overall product only six digits long. F = 8 yields a seven-digit product.

2 Paul is married to Ursula, Quentin is married to Sue, and Ron is married to Tess. Since each person bought a number of stamps equal to the value of one of the stamps he or she bought, the amount of money each spent was a square number. Therefore, $h_i^2 - w_i^2 = (h_i - w_i)(h_i + w_i) = 45$ for $i = 1, 2, 3$. Now, 45 can be factored into two factors in three ways: 1x45, 3x15, and 5x9. Setting $h_i - w_i$ to the smaller factor and $h_i + w_i$ to the larger factor and solving gives $h_1 =23$, $w_1 = 22$; $h_2 = 9$, $w_2 = 6$; and $h_3 = 7$, $w_3 = 2$. It is now easy to deduce the above pairings.

3 A circle with radius $R=\sqrt{(4-2\sqrt{2})}/2 = 0.541196$ will minimize the area included in either the unit square or the circle, but not common to both, when their centers are coincident. In the accompanying figure, $A_1 + A_2 = (\alpha/2\pi)(\pi R^2) = \alpha R^2/2$; $A_2 = (L/2)(L\tan\alpha) = L^2\tan\alpha/2$; $A_2 + A_3 + A_4 = L^2/2$; and $A_3 = (\beta/2\pi)(\pi R^2) = \beta R^2/2$. From this, we find $A_1 + A_4 = R^2(\alpha-\beta)/2 - L^2\tan\alpha + L^2/2$; but $\alpha-\beta = 2\alpha-\pi/4$, and $R = L\sec\alpha$. Therefore, $A_1 + A_4 = \alpha L^2\sec^2\alpha - \pi L^2\sec^2\alpha/8 - L^2\tan\alpha + L^2/2$. Differentiating with respect to α and setting the differential equal to 0 give $2\alpha L^2\sec^2\alpha\tan\alpha - \pi L^2\sec^2\alpha\tan\alpha/4 = 0$. Thus, $\alpha = \pi/8$, and, since $L = 0.5$, $R = 0.5/\cos(\pi/8) = 0.541196$.



4 A probability $p = 0.00465242$ will assure one chance in a million of firing on a friendly aircraft. Let p equal the desired probability. Then, the probability that all five channels fail is p^5; the probability that four channels will fail is $C(5, 4)p^4(1 - p)$; and the probability that three channels will fail is $C(5, 3)p^3(1 - p)^2$, where $C(n, m)$ is the combination of n things taken m at a time. Summing these probabilities gives $p^5 + 5p^4(1 - p) + 10p^3(1 - p)^2 = 6p^5 - 15p^4 + 10p^3 = 10^{-6}$, which leads to the solution given above.

5 John was told he would be discharged between 4 and 5 days. He was released at 4 days with a reading of 4.9625 mrem. For a decay process, $m = m_0 e^{-kd}$. Therefore, $k = \ln(m_0/m)/d$. If h is the half-life of the process, $k = \ln 2/h$, or $m = m_0 2^{-d/h}$. For I_{131}, the half-life is 8.05 days, and John's half retention time was 0.84 days. With two decay processes, the net result is their product. Hence, $m = 190(2^{-d/8.05}\, 2^{-d/0.84})$. Letting d vary from 1 through 5 shows that at 4 days, $m = 4.9625$ mrem. Using a half retention time of 0.75 gives $m = 3.34$ mrem in 4 days, and a half retention time of 1.0 gives 3.86 mrem in 5 days.

Bonus The times when the hour, minute, and second hands are closest to trisecting an analog twelve-hour clock's face are 2:54:34.55 and 9:05:25.45. Let $t = h{:}m{:}s$. Then, $\theta_h = 30(h + m/60 + s/3600)$; $\theta_m = 6(m + s/60)$; and $\theta_s = 6s$, where θ_h is the angle of the hour hand, θ_m the angle of the minute hand, and θ_s the angle of the second hand. Let $A = \theta_h - \theta_m = 30h - 11m/2 - 11s/120$; $B = \theta_m - \theta_s = 6m - 59s/10$; and $C = \theta_h - \theta_s = 30h + m/2 - 719s/120$. One approach is to look for times when $A = 120°$ and then see for which of these times B and C are closest to 120° (there are 24 such times). The best times are those given above.

Computer Bonus The next weak prime after 294,001 is 505,447. You can email *dondechman@aol.com* if you would like a copy of his computer program, written in Basic language, that solves this problem.

NEW FALL PROBLEMS

1 At the end of the soccer season, it was found that the total number of goals scored by each of the 11 players on the team was a prime number and that the average was also a prime number. Furthermore, no two players had scored the same number of goals, nor had anyone scored the same as the average. If no one scored more than 45 goals, how many goals did each player score?

—*Nearly Impossible Brain Bafflers* by Tim Sole and Rod Marshall

2 What is the exact probability that a randomly selected month (in the Gregorian calendar) will have five Sundays?

—**Howard G. McIlvried,** *PA Γ '53*

3 Four men and their wives are scheduled for a mixed-doubles tennis tournament on two adjacent courts for three consecutive time periods. Everyone plays three matches, each with a different partner of the opposite sex and with no repeated opponent. No one ever plays either with or against his or her own spouse. How many possible tournaments are there, ignoring permutations of courts and time slots?

—**Byron R. Adams,** *TX* Α *'58*

4 Marrieds are complete liars in the village of Loose Chippins, whereas singles are wholly truthful. Take that group of three men and three women over there, for instance. They are two couples and two singles. If you asked Charlie whether Bert is married to Queenie, he would say "Urp." If you asked Pauline if Pauline is married to Charlie, she would say "Urp." If you asked Alf if Alf is married to Rhona, he would say "Ong." And if you knew enough of the local dialect to know whether "Urp" means "Yes" and "Ong" means "No," or vice versa, you could work out who is married to whom. Who is married to whom?

—*New Scientist*

5 TWO + TWENTY = TWELVE + TEN, with the first three divisible by their namesakes. Note: different letters stand for different digits, and same letter is always the same digit.

—*Technology Review*

Bonus On what day of the year is twilight shortest at a place of given latitude at sea level? Express your answer in the form: March 21st ± f(latitude). Assume the inclination of the ecliptic is 23° 27′. Assume that the angular distance of the sun from the equinox (assumed to be noon March 21 for this problem) changes at an average daily rate of 360°/365.25. Assume twilight starts the moment at which the midpoint of the sun is intersected by the horizon. Assume twilight ends when the midpoint of the sun stands 6.5 degrees below the horizon; this is civil twilight. Ignore effects of atmospheric refraction. As a check of your math, the shortest twilight in Leipzig, Germany, (51° 20′ 6″ N latitude) is when?

—*100 Great Problems of Elementary Mathematics* by Heinrich Dörrie

Double Bonus Given the base and altitude of a triangle, with ruler and compasses, find the length of a side of the inscribed square.

—Alsop, 1848

Send your answers to any or all the Fall Brain Ticklers to:
Jim Froula, Tau Beta Pi, P.O. Box 2697, Knoxville, TN 37901-2697.
If your answers are plain text only (no HTML, no attachments), email them to *BrainTicklers @tbp.org*. The cutoff date is the appearance of the Winter Bent in early January. Acknowledgments of your entry will be made in the Spring '05 column. The method of solution is not necessary, unless you think it may be of interest to the judges. We welcome any interesting new problems that may be suitable for use in this column. The Double Bonus is not graded. Jim will forward your entries to the judges:
H.G. McIlvried III, *PA Γ '53,*
D.A. Dechman, *TX* Α *'57,*
J.L. Bradshaw, *PA* Α *'82,*
and the columnist for this issue,
—**F.J. Tydeman,** *CA Δ '73.*