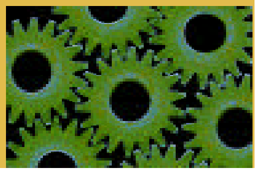

Brain Ticklers

RESULTS FROM SPRING 2003

Perfect

* Capelli, Ronald B.	MI	Γ	'73
* Couillard, J. Gregory	IL	A	'89
* Garnett, James M.	MS	A	'65
* Kimsey, David B.	AL	A	'71
* Mayer, Michael A.	IL	A	'89
* Riedesel, Jeremy M.	OH	B	'96
* Schmidt, V. Hugo	WA	B	'51
* Thaller, David B.	MA	B	'93
* Verser, Rocke C.	KS	Γ	'77
* White, R. Dudley	VA	A	'76

Other

* Alderson, William S.	MI	E	'43
* Anderson, Richard R.	IL	Γ	'49
Anderson, Paul MacLean	Son of member		
* Anidi, Chibueze E.	Non-member		
* Aron, Gert	IA	B	'59
* Baines, Elliot A., Jr.	NY	Δ	'78
* Barker, Calvin L. R.	TX	A	'53
Barr, Adam D.	NJ	Δ	'88
* Barthel, Gerald R.	OH	B	'67
* Bernacki, Stephen E.	MA	A	'70
* Bernheisel, Jay D.	IN	B	'96
* Biggadike, Robert H.	AR	A	'58
* Brule, John D.	MI	B	'49
* Brzezinski, Mark A.	OH	B	'00
* Cameron, Charles M.	OH	Γ	'01
* Creutz, Michael J.	CA	B	'66
Creutz, Edward C.	PA	Γ	'36
* Galer, Craig K.	MI	A	'77
* Garside, Jeffrey J.	WI	E	'90
* Gifford, William C.	AR	A	'64
Carman, Michael S.	Non-member		
* Griggs, James L., Jr.	OH	A	'56
* Hales, Jason D.	UT	B	'97
Jones, Donlan F.	CA	Z	'52
* Kay, Nathaniel E.	OH	Λ	'01
Lalinsky, Mark A.	MI	Γ	'77
Lew, Thomas M.	TX	Δ	'84
Marrone, James D.	IN	A	'87
Marrone, James I.	IN	A	'61
* Midgley, James E.	MI	Γ	'56
Miller, Keith A., Jr.	PA	Λ	'81
* Mitchell, Donald B.	CA	Γ	'59
* Mrozek, Paul M.	MI	H	'91
* Nabutovsky, Joseph	Father of member		
* O'Keeffe, Claudia Marx	MD	B	'92
* Pecsvaradi, Thomas	PA	Z	'64
* Peithman, Harlan W., Jr.	IL	A	'53
* Quintana, Juan S.	OH	Θ	'62
Rasbold, J. Charles	OH	A	'83
Rentz, Peter E.	IN	A	'55
* Rieger, Paul J.	KY	A	'66
Robert, Christopher A.	IA	A	'01
* Routh, Andre G.	FL	B	'89
Small, Mitchell J.	NY	Γ	'61
* Snyder, M. Duane	IA	B	'63
* Spong, Robert N.	UT	A	'58
* Stribling, Jeffrey R.	CA	A	'92
* Strong, Michael D.	PA	A	'84
* Thiele, Karl E.	NY	Γ	'82
Valko, Andrew G.	PA	Λ	'80
* Van Sickel, James R., Jr.	OH	H	'77
* VanShaar, Steven R.	UT	Γ	'00
* Wolff, Nicholas L.	NE	A	'00
* Yatchman, Michael J.	MI	Γ	'80
Yee, David G.	NJ	B	'04

* Denotes correct bonus solution

R.W. "Bill" Rowland, *MD* B '*51*, the oldest ever Brain Tickler judge, is calling it quits and retiring after 20 plus years of faithful service. **John L. Bradshaw,** *PA* A '*82*, a long-time solver of Brain Ticklers, has agreed to take his place. John will be writing the Summer columns from now on.

SPRING REVIEW

Problem 2 about the time for the three planets to realign was the hardest, with less than 1/3 of the submitted answers being correct. On the other hand, the Bonus was too easy, as it had the highest percentage of correct answers of all the problems. Thomas Pecsvaradi, *PA* Z '*64*, said his friend Giovanni Vannucci found a much smaller answer to the Spring Computer Bonus. If you explore bases other than 10, he found that in base 31, 1HH1 has a cube root of 15_{31}, a smaller number than the answer of 2201_{10} that we provided.

SUMMER ANSWERS

Readers' entries for the Summer problems will be acknowledged in the Winter BENT. Meanwhile, here are the answers:

1. You were to find the formula (in terms of the number n of balls on one edge) for the total number of cannonballs in a triangular-base-pyramid pile. By the use of finite difference techniques or simultaneous equations, $n(n+1)(n+2)/6$ is the asked for formula. Also, by inspection, the number of cannonballs in each layer, starting at the top, is 1, 3, 6, 10, ... These are the so-called triangular numbers, given by $T_i = i(i+1)/2 = (i^2 + i)/2$. Therefore, the total number of cannonballs is ΣT_i from $i = 1$ to $i = n$, which equals $1/2\ \Sigma i^2$ from $i = 1$ to $i = n$ plus $1/2\ \Sigma i$ from $i = 1$ to $i = n$. From a table of sums of powers of integers, we find that $\Sigma i^2 = n(n+1)(2n+1)/6$ and $\Sigma i = n(n+1)/2$. Adding these together and dividing by 2 gives the answer of $n(n+1)(n+2)/6$.

2. This problem asked for the probability that three different random numbers, from 1 to 10, could indicate the side lengths of a triangle. There are $C(10, 3) = 120$ combinations of 10 balls/numbers drawn 3 at a time. Call the numbers x, y, z from smallest to largest. The triangle requires that $x + y > z$. Count the ways to satisfy this inequality for each z. For $z = 4, 5, 6, \ldots, 10$, the ways are 1, 2, 4, 6, 9, 12, 16 for a total of 50. Thus, the desired probability is 50/120 = 5/12.

3. FIVE – FOUR = ONE
FIVE + FIVE = EVEN
2 3 7 4 – 2 1 9 0 = 1 8 4
2 3 7 4 + 2 3 7 4 = 4 7 4 8

4. This problem involves a hole of radius R along the diagonal of a one-meter cube, which has a diagonal length of $\sqrt{3}$. You can visualize the shape better if you make a model of the cube/cylinder relationship. Consider a right-circular cylinder of diameter R and height $\sqrt{3}$ oriented along the diagonal of the cube. This cylinder can be considered as consisting of the hole through the cube plus volume outside the cube at both ends of the cylinder. This extended-hole volume can be visualized as consisting of 12 figures of equal volume, V_S, six identical and six mirror images. The hole volume is $V = \pi R^2 \sqrt{3} - 12V_S$. Each dV_S is a rectangular-base pyramid with height R, width $R\, d\varphi$, and length $R \sqrt{2} \cos\varphi$, where φ is the angle around the hole axis. Then $dV_S = (R^3 \sqrt{2}/3) \cos\varphi\, d\varphi$. Integrating this from 0 to 60° gives $V_S = R^3/\sqrt{6}$ and $V = \pi R^2 \sqrt{3} - 2R^3 \sqrt{6}$. Finally, for $R = 0.125$, the desired hole volume is $V = 0.075454\ \text{m}^3$.

5. The Earth's rotation rate relative to the fixed stars was requested. A year is 365 + 97/400 = 365.2425 days. In a year the Earth has 1 + 365.2425 rotations. Thus the rotation rate is 360(366.2425/365.2425) = 5,859,880/16,233 = 360.985647 degrees per day.

BONUS. This is the one about the cone with a length-plus-girth of 108 inches. There are two segments of girth G – one (G_1) around the conical surface and the other (G_2) across the cone's base. $G = 2(G_1+G_2)$. The conical segment is not an ellipse (as would result from a plane intersecting the cone), but rather is a straight line when the cone is flattened.

Let L, r, and θ be the slant height, radius, and half-angle of the cone with $r = L \sin\theta$. The flattened half-cone is a circle sector with radius L and total angle $\pi r/L$. Taking α on this sector as the angle from top to the intersection of G_1 and G_2, the relationship is $G_1 = L \sin(\pi r/L - \alpha)$. On the cone's base, take β as the angle from top to the G_1/G_2 intersection. Then $G_2 = r \sin\beta$ and $\alpha/\beta = r/L$. Using these equations and setting $dG/d\beta = 0$ leads to $\beta = \pi/(1 + L/r)$. Trials with different θ show that maximum volume occurs for θ = 30° and β = 2α = 60°. This satisfies intuition because the cone's diameter and slant height equally represent the maximum length at this θ. Utilization of the $L + G = 108$ condition leads to the final results of $G = 77.984$, diameter = $2r = L = 30.016$, altitude = 25.995, and volume = 6,131.4, all in inch units.

Construct a cone, and slide a loop of string over it, and you will be able to visualize the solution.

A similar problem last Winter used different definitions of girth and length to achieve a volume of 4,950.4.

COMPUTER BONUS. A computer search shows that there are 87 ten-digit perfect squares that use each of the digits. The smallest and largest are $32{,}043^2$ = 1,026,753,849 and $99{,}066^2$ = 9,814,072,356.

NEW FALL PROBLEMS

1. It was Mycroft's wont to send Sherlock little problems by telegram from time to time. This morning's read: "Anstey, Buchan, Collins, and Doyle played a 3-round singles tournament (2 matches per round). They finished in alphabetical order by name. Scoring was win 2, draw 1, loss 0. Lion tamer won, and Musician drew in the first round. Neurologist won in last round. Total number of draws was. What is name and final score of Ornithologist?"

"But Holmes," I protested, "he has omitted the total number of draws."

"Or else the post office has," remarked my friend with a flash of his piercing intellect.

"So the problem must be unsolvable."

"By no means, my good Watson. He plainly intended to put it in. Or, if he did not, he plainly intended us to think he intended to put it in. And, as miserliness restrains him from telegraphing needless data, whereas pride prevents him sending insufficient data, it is as if he had sent it."

What is the answer to Mycroft's question?

—Martin Hollis

2. Three friends, Ann, Betty, and Carol, meet to celebrate the birthday of one of them. "How old are you now, Ann?" asks Betty. "Well," says Ann, "using our exact (not rounded) ages, the sum of our three ages is 80 years. I am twice as old as Carol, and at the time when I become twice as old as you, our three combined ages will be half again as large as now." The question is, whose birthday is it?

—The Platinic Corner

3. Al and Ben play a game involving a series of coin tosses. Each gambler picks a different series of three possible outcomes, and the winner is the one whose chosen series of outcomes first occurs in the string of coin tosses. Al announces that his series is HHH. If Ben makes an optimal choice for his series, what is his probability of winning?

— *nearly impossible Brain Bafflers* by Tim Sole and Rod Marshall

4. How fast, that is, how many rotations per year, would the Earth have to rotate in order for a 75 kg person at the equator to lift off the ground? Ignore atmospheric effects.

—**Walter O. Stadlin,** *NJ* Γ *'52*

5. A right-circular cone of height H and radius R is rolled on a plane so that the apex remains at a fixed point. How many times will the cone revolve about its axis if the cone is rolled through a complete circle on the plane?

—Daryl Cooper

BONUS. You have a large supply of one-ohm resistors and some wire, and you wish to create a "black box," i.e., a box with only several terminals showing, which will allow you to achieve any integral resistance from 1 through 10 ohms, inclusive, by proper choice of two of the terminals. How should such a box be wired (once wired, the wiring is fixed) if the number of terminals is a minimum? Using some wire to short terminals is not allowed. Remember, resistance can be reduced by wiring resistors in parallel. The solution is relatively simple and does not require any complicated wiring scheme.

—**Hubert W. Hagadorn,** *PA* Ε *'59*

COMPUTER BONUS. For all values of N greater than 4, the decimal representation of $N!$ (N factorial) ends with a string of zeros. We are interested in the digit preceding these zeros. What is the value of the non-zero digit immediately preceding the string of zeros at the end of 1,000,000!?

—**Howard G. McIlvried III,** *PA* Γ *'53*

Postal mail your answers to any or all of the Brain Ticklers to:

Jim Froula
Tau Beta Pi
P. O. Box 2697
Knoxville, TN 37901-2697

or email plain text (no HTML, no attachments) to:

BrainTicklers@tbp.org

The cutoff date for entries to the Fall column is the appearance of the Winter Bent around mid-January. The method of solution is not necessary, unless you think it will be of interest to the judges. We also welcome any interesting new problems that may be suitable for use in the column. The Computer Bonus is not graded. Jim will forward your entries to the judges, who are:

H.G. McIlvried III, *PA* Γ *'53,*
D.A. Dechman, *TX* Α *'57,*
J. L. Bradshaw, *PA* Α *'82,*
and the columnist for this issue,
F.J. Tydeman, *CA* Δ *'73.*