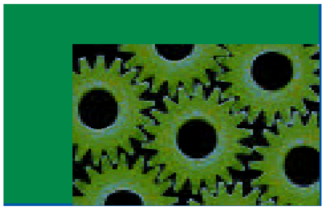


50 Years of Brain Ticklers

SPRING REVIEW

Of the regular Spring problems, the most difficult was No. 4 about the 1,234,567,890th permutation of letters of the alphabet. The Bonus also proved to be moderately difficult, with only about one third of the entries containing a correct solution.

Although the answer of 9 to Spring No. 1 given in the Summer column is correct, there is an error in the logic that was provided. Delete the two sentences, "Therefore, N has to be 5, . . . clarifies." and replace with, "Thus, N is a nonsquare. Now, if N is divisible by 3, D's answer would not help. Therefore, N is not divisible by 3 and must be 5, 7, or 8. After D's answer, R concludes that N is 7 or 8, which P's answer clarifies."

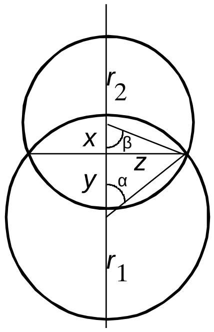
SUMMER ANSWERS

Readers' entries for the Summer problems will be acknowledged in the Winter BENT. Meanwhile, here are the answers:

1. You were to find the probability that the golden anniversary of a date in the period from 1900 to 1951 fell on the same day of the week as the original event. For the years 1948 to 1951, the number of months having anniversary matches are 2, 0, 10, and 12, respectively, which translates to 730 days having matches out of a total of 1,461 days. The pattern repeats every four years except when disturbed by the year 1900, which was not a leap year, so that the 59 days in January and February, 1900, do not have matches. Therefore, the probability that, for an event that occurred in the 52-year span from 1900 to 1951, the fiftieth anniversary fell on the same day of the week is P = (13 × 730-59)/(13 × 1,461-1) = **9,431 / 18,992** = 0.49658. Incidentally, the corresponding probability for the 400-year cycle of the Gregorian calendar is 27,022 / 146,097 = 0.18496.

2. THREE+THREE+FIVE=ELEVEN is 56,711 + 56,711 + 8,491 = 121,913, where ELEVEN is divisible by 11. E must be 1 or 2. If E =1, then N = 3 and V = 9. If E = 2, then N = 6 and V = 8. For ELEVEN to be divisible by 11, ELEVEN must be 121,913 or 232,826. But 232,826 is eliminated since even T= 9 can't yield 23x,xxx. The only combinations of R and I are (0,8), (4,0), and (7,4); and (7,4) is the only combination that yields an answer.

3. The problem was to find the distance between the centers of two circles in a Venn diagram. The relative areas of the larger circle, smaller circle, and overlap are 100, 71, and 33, respectively. The larger circle has r_1=5 cm, giving $r_2=(\sqrt{71})/2$ cm. Then $y = r_1 \cos\alpha$, $z=r_1 \sin\alpha$, $x^2+z^2=r_2^2$, $\tan\beta = z/x$, $A_1=(\alpha r_1^2 - yz)$, $A_2=(\beta r_2^2 - xz)$, and $(A_1+A_2)(4/\pi)= 33$, where A_1 is the area of the larger circle segment and A_2 is the area of



the smaller circle segment. The seven equations with seven unknowns can be solved by successive approximations of alpha to give the desired center-to-center distance of $x+y = 4.598$ cm.

4. For the first logistician (Log 1) to know the word, his letter had to be unique. That eliminates the word TOE. Log 2's letter had to be unique among the remaining words, thus eliminating the word OAR. Log 3's letter had to be the middle one in HOE, PAD, or VAT. So the magic word is **HOE**.

5. The smallest positive integer that has at least 1,000 different integral factors is 245,044,800. An integer may be expressed as a product of its prime factors: $N = A^a B^b C^c \ldots$. The number of factors is $F = (1+a)(1+b)(1+c) \ldots$. For smallest N value, $A, B, C, \ldots$ must be the small primes (2, 3, 5, . . .). The approach is to factor the numbers 1,000, 1,001, 1,002, . . . looking for numbers with all small prime factors. The three most promising are $1{,}000 = 2^3 5^3$, $1{,}008 = 2^4 3^2 7$, and $1{,}024 = 2^{10}$. Of these, 1,008 gives $N = 2^6 3^2 5^2 7 \times 11 \times 13 \times 17 =$ 245,044,800, which is the smallest.

Bonus. You were to find the common volume for a horizontal cylinder intersecting a vertical cylinder of twice its diameter. Let the x and z axes be along the centerlines of the smaller and larger cylinders and r be the radius of the smaller cylinder. The volume is given by $V = 8\int_0^r x\, z\, dy$ where $z = \sqrt{(r^2 - y^2)}$ and $x = \sqrt{(4r^2 - y^2)}$. Then, for $r = 0.5$ m, numerical integration gives $V = 1.52004$ m^3.

Computer Bonus. The largest four-digit number whose square contains only digits in the original number is $4{,}832^2 = 23{,}348{,}224$.

NEW FALL PROBLEMS

1. What is the total number of pips on the 136 dominoes making up a double-fifteen set?

—*Madachy's Mathematical Recreations* by Joseph S. Madachy

2. Solve the following cryptic multiplication: PEN × INK = LETTER with different letters being different digits, the same letter being the same digit throughout, and no leading zeros.

—Howard G. McIlvried, *PA* Γ *'53*

3. In a game of bridge, South is declarer at a contract of three no trump and holds A, K, 10, x, x of spades, while dummy holds J, x, x of spades. (Declarer has no knowledge of how the rest of the spade suit is distributed.) After winning the first trick, on which no spades were played, declarer leads the A of spades. Both opponents follow suit, but the Q does not drop. What is the probability that the Q will fall if declarer now leads the K of spades?

—John W. Langhaar, *PA* A *'33*

5. What is the maximum number of knights that can be placed on a standard 8x8 chess board, so that no knight threatens another knight, i. e., no knight can move to a square occupied by another knight? Also, what is the placement of the knights on the board? A knight moves two squares in one direction and one square in another direction to end up on the diagonally opposite corner of a 2 × 3 grid. The move can occur even if intervening squares are occupied.

—Daryl Cooper

Bonus. Most people are familiar with how water sloshes back and forth in a bathtub, rising and falling at the ends of the tub, similar to ocean tides, but on a much higher frequency. A similar phenomenon can occur in a lake, usually initiated due to the action of the wind piling up water at one end. Assume the Lake of Geneva behaves like a tank of water, 70 km long, 8 km wide, and 150 m deep. What is the period of the unimodal oscillations in the lake?

—*Vibrations and Waves* by A.P. French

Computer Bonus. Consider the equation: $x^7 + y^3 = z^2$. Two solutions to this equation are: $x = 1, y = 2, z = 3$; and $x = 2, y = 17, z = 71$. Find another solution in which x, y, and z are relatively prime positive integers.

—*Mathematical Mountaintops* by John L. Casti

The judges are:
H.G. McIlvried III, *PA* Γ *'53;*
R.W. Rowland, *MD* B *'51;*
D.A. Dechman, *TX* A *'57;* and
the columnist for this issue,

—F.J. Tydeman, *CA* Δ *'73.*